Inclusive diffraction in Nuclei

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Abstract

We calculate nuclear diffractive structure functions at small $x_{\mathbb{P}}$ in the IPsat (Kowalski-Teaney) model. This model has previously been shown to provide good agreement with inclusive F_2 measurements and exclusive vector meson measurements at HERA. We discuss how the impact parameter dependence crucially affects our analysis, in particular for small β.

Outline

- Inclusive and diffractive DIS in dipole model
- Saturation scale, A-dependence (IPsat model)
- Q_s in the MV model
- Diffractive structure functions
  - computing
  - results vs. β, Q^2, $x_\mathbb{P}$, A.

Talk based on:

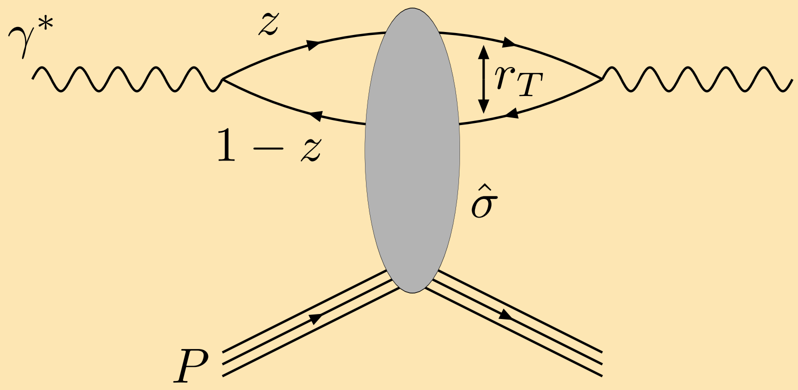
- H. Kowalski, T. Lappi, C. Marquet and R. Venugopalan, arXiv:0805.4071 [hep-ph].

To a lesser extent

- H. Kowalski, T. Lappi and R. Venugopalan, Phys. Rev. Lett. **100**, (2008) 022303, arXiv:0705.3047 [hep-ph].
- T. Lappi, Eur. Phys. J. C **55** (2008) 285 [arXiv:0711.3039 [hep-ph]].

Dipole cross section

DIS at high energy/small x: dipole cross section, can be calculated from the dominant component of classical field:



$$\hat{\sigma}(\mathbf{r}_T) = \int \mathrm{d}^2\mathbf{b}_T \frac{1}{N_\mathrm{c}} \mathrm{Tr} \left\langle 1 - U^\dagger\left(\mathbf{b}_T + \frac{\mathbf{r}_T}{2}\right) U\left(\mathbf{b}_T - \frac{\mathbf{r}_T}{2}\right) \right\rangle$$

$$U(\mathbf{x}_T) = P \exp\left\{ ig \int \mathrm{d}x^- A^+_{\mathrm{cov}}(\mathbf{x}_T, x^-) \right\}$$

This same Wilson line gives the (LC gauge) pure gauge field in the $\tau = 0$ initial condition for the Glasma description of nucleus-nucleus collisions.

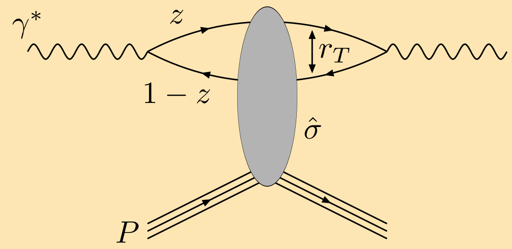
$$A^i_{\text{(one nucleus)}} = \frac{i}{g} U(\mathbf{x}_T) \partial_i U^\dagger(\mathbf{x}_T)$$

Basics: small-x and dipole cross section

$$\sigma_{\text{dip}}(x,\mathbf{r}_T) = \int \mathrm{d}^2\mathbf{b}_T \frac{\mathrm{d}^2\sigma_{\text{dip}}(x,\mathbf{r}_T,\mathbf{b}_T)}{\mathrm{d}^2\mathbf{b}_T}$$

$$\sigma_{\text{dip}}(x,\mathbf{r}_T,\mathbf{\Delta}) = \int \mathrm{d}^2\mathbf{b}_T \frac{\mathrm{d}^2\sigma_{\text{dip}}(x,\mathbf{r}_T,\mathbf{b}_T)}{\mathrm{d}^2\mathbf{b}_T} e^{i\mathbf{b}_T\cdot\mathbf{\Delta}},$$

$$\mathbf{\Delta}^2 = -t$$



Total $\gamma^* p$: $\sigma_{L,T}^{\gamma^* p} = \int \mathrm{d}^2\mathbf{r}_T \int \mathrm{d}z \left|\Psi^{\gamma}_{L,T}(Q^2,\mathbf{r}_T,z)\right|^2 \sigma_{\text{dip}}(x,\mathbf{r}_T)$

Total diff.: $\frac{\sigma^D_{L,T}}{\mathrm{d}t} = \frac{1}{16\pi}\int \mathrm{d}^2\mathbf{r}_T \int \mathrm{d}z \left|\Psi^{\gamma}_{L,T}(Q^2,\mathbf{r}_T,z)\right|^2 \sigma^2_{\text{dip}}(x,\mathbf{r}_T,\mathbf{\Delta})$

X-cl. diff.: $\frac{\sigma^D_{L,T}}{\mathrm{d}t} = \frac{1}{16\pi}\left|\int \mathrm{d}^2\mathbf{r}_T \int \mathrm{d}z \left(\Psi^{\gamma}\Psi^{*V}\right)_{L,T} \sigma_{\text{dip}}(x,\mathbf{r}_T,\mathbf{\Delta})\right|^2$

Use:

- S-matrix real
- optical theorem

- $\Psi^{\gamma}(Q^2,\mathbf{r}_T,z) \sim K_{0,1}(\sqrt{z(1-z)}Q|\mathbf{r}_T|)$ ▶ momentum scale $Q^2 \sim 1/\mathbf{r}_T{}^2$
- Diffractive: t distribution is FT of $\mathbf{b}_T$ distribution.

IPsat model: protons

GBW[1]: $\sigma_{\text{dip}} = \sigma_0(1 - e^{-{\mathbf{r}_T}^2/Q_{\text{s}}(x)^2})$ ▶ improvements by many authors ▶ use here the IPsat model by Kowalski, Teaney[2,3]

- DGLAP evolution: improves description large Q^2.
- Consistently use same impact parameter dependence for both total and diffractive cross sections

$$\sigma_{\text{dip}}(x, \mathbf{r}_T) = 2 \int \mathrm{d}^2\mathbf{b}_T \left(1 - \exp \overbrace{\left\{ -\frac{\pi^2}{2N_{\text{c}}} \alpha_{\text{s}}(\mu^2) x g(x, \mu^2) T_p(\mathbf{b}_T) {\mathbf{r}_T}^2 \right\}}^{-{\mathbf{r}_T}^2 Q_{\text{s}}^2/4} \right),$$

with $\mu^2 = \frac{C}{{\mathbf{r}_T}^2} + \mu_0^2$, Gaussian $T_p(\mathbf{b}_T)$

$xg(x, \mu^2)$ is evolved with DGLAP, initial condition $A_g x^{-\lambda_g}(1-x)^{5.6}$
Best fit result $\lambda_g < 0$: increase of $Q_{\text{s}}^2 \sim x^{-\lambda}$ comes entirely from DGLAP!

[1] K. Golec-Biernat and M. Wusthoff, *Phys. Rev.* **D59** (1999) 014017 [hep-ph/9807513].
[2] H. Kowalski and D. Teaney, *Phys. Rev.* **D68** (2003) 114005 [hep-ph/0304189].
[3] H. Kowalski, L. Motyka and G. Watt, *Phys. Rev.* **D74** (2006) 074016 [hep-ph/0606272].

IPsat: nuclei[2,4]

Straightforward generalization to nuclei:

$$\frac{\mathrm{d}\sigma^A_{\mathrm{dip}}}{\mathrm{d}^2\mathbf{b}_T} = 2\left(1 - \exp\left\{-\frac{\pi^2}{2N_\mathrm{c}}\alpha_\mathrm{s}(\mu^2)xg(x,\mu^2)\sum_{i=1}^{A} T_p(\mathbf{b}_T - \mathbf{b}_{Ti})\mathbf{r}_T{}^2\right\}\right),$$

$\mathbf{b}_{Ti}$: nucleon positions, from Woods-Saxon, average $\langle\cdot\rangle_\mathrm{N}$

Proton radius $\sim 0.6\mathrm{fm}$, much less than the nucleon-nucleon distance.
▶ **lumpy** nucleus ▶ visible in incoherent diffraction $\left\langle {\sigma^A_{\mathrm{dip}}}^2 \right\rangle_\mathrm{N}$

$$\left\langle \frac{\mathrm{d}\sigma^A_{\mathrm{dip}}}{\mathrm{d}^2\mathbf{b}_T} \right\rangle_\mathrm{N} = 2\left[1 - \left(1 - \frac{T_A(\mathbf{b}_T)}{2}\sigma^p_{\mathrm{dip}}\right)^A\right] \approx_{A\to\infty} 2\left[1 - e^{-\frac{AT_A(\mathbf{b}_T)}{2}\sigma^p_{\mathrm{dip}}}\right]$$

($\int \mathrm{d}^2\mathbf{b}_T T_A(\mathbf{b}_T) = 1$ ▶ $T_A(\mathbf{0}_T) \sim A^{-2/3}$)

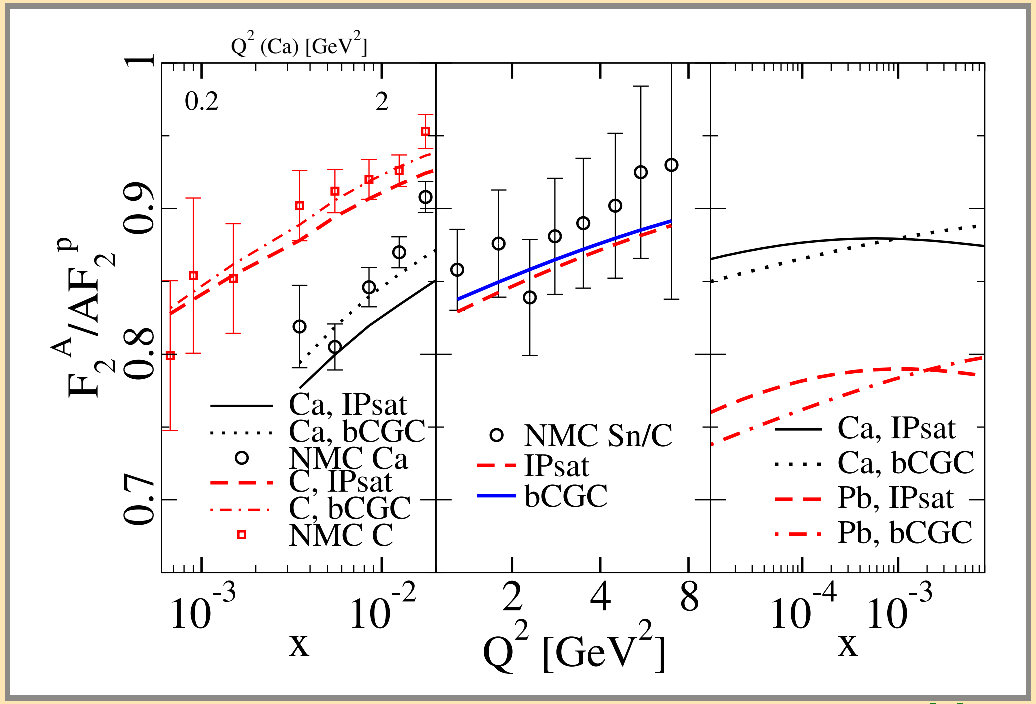
[2] H. Kowalski and D. Teaney, *Phys. Rev.* **D68** (2003) 114005 [hep-ph/0304189].
[4] H. Kowalski, T. Lappi and R. Venugopalan, *Phys. Rev. Lett.* **100** (2008) 022303 [0705.3047].

Inclusive eA with IPsat

Good description of existing (very limited) small-x eA data with **no nuclear fit parameters**:

Only thing needed

- IPsat parameters fit to ep
- standard Woods-Saxon



Also shown bCGC: b-dependent version [3] of IIM[5] parametrization.

[3] H. Kowalski, L. Motyka and G. Watt, *Phys. Rev.* **D74** (2006) 074016 [hep-ph/0606272].

[5] E. Iancu, K. Itakura and S. Munier, *Phys. Lett.* **B590** (2004) 199 [hep-ph/0310338].

A-dependence of Q_{s}: previous estimates

Earlier fits of eA data:

Freund, Rummukainen, Weigert[6]: result $Q_{\mathrm{s}}^2 \sim A^{1/4}$ (but with $R_A^2 = A^{2/3+0.1} R_p^2$) ▶ RHIC energy $Q_{\mathrm{s}} \approx 1.4\mathrm{GeV}$ (adj)

Armesto, Salgado, Wiedemann [7]:

$$Q_{\mathrm{s}}^{A\,2} = \left(\frac{A\pi R_p^2}{\pi R_A^2}\right)^{\alpha} (Q_{\mathrm{s}}^p)^2 \text{ with } R_A = (1.12A^{1/3} - 0.86A^{-1/3})\mathrm{fm}.$$

Fit R_p and α

▶ **Statement: "Favors $Q_{\mathrm{s}}^{A\,2} \sim A^{4/9}$"** (but this is asymptotic)

▶ RHIC energy $Q_{\mathrm{s}} \approx 1.3\mathrm{GeV}$ (adj)

[6] A. Freund, K. Rummukainen, H. Weigert and A. Schafer, *Phys. Rev. Lett.* **90** (2003) 222002 [hep-ph/0210139].

[7] N. Armesto, C. A. Salgado and U. A. Wiedemann, *Phys. Rev. Lett.* **94** (2005) 022002 [hep-ph/0407018].

$A^{1/3}$ from the IPsat model?

$$\frac{\mathrm{d}\sigma^A_{\mathrm{dip}}(x,\mathbf{r}_T,\mathbf{b}_T)}{\mathrm{d}^2\mathbf{b}_T} \approx 2\left[1 - e^{-\frac{A T_A(\mathbf{b}_T)}{2}\sigma^p_{\mathrm{dip}}}\right]$$

Q_s is not directly a parameter. Define

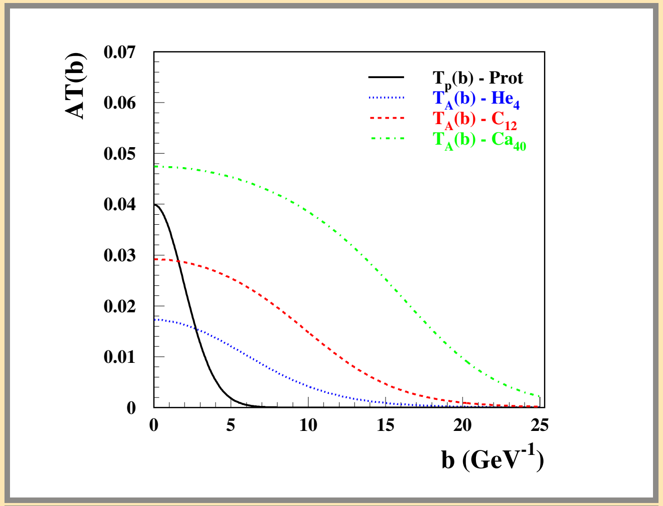
$$\frac{\mathrm{d}^2\sigma_{\mathrm{dip}}(x,\mathbf{r}_T,\mathbf{b}_T)}{\mathrm{d}^2\mathbf{b}_T} = 2(1-e^{-1/4}),$$

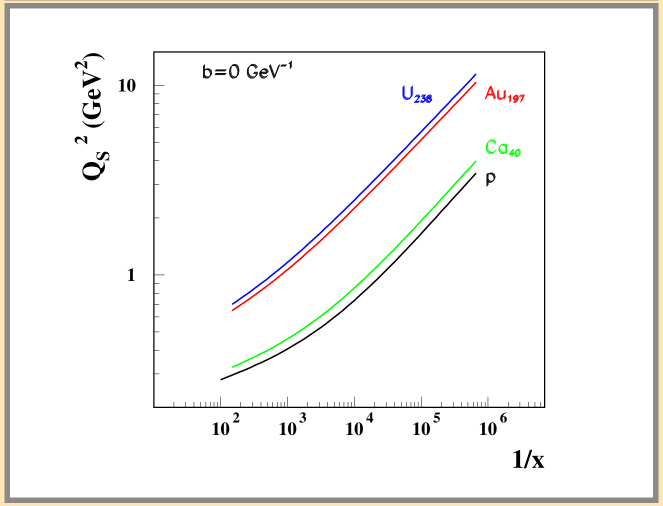
when ${\mathbf{r}_T}^2 = 1/Q_\mathrm{s}^2$.
If we simplify this by approximating $\sigma^p_{\mathrm{dip}}(\mathbf{r}_T) = 2\pi R_p^2 e^{-{\mathbf{r}_T}^2 (Q_\mathrm{s}^p)^2/4}$ and $T_A(\mathbf{b}_T) = \theta(R_A - |\mathbf{b}_T|)/(\pi R_A^2)$

- for $A \gg 1$

$${Q_\mathrm{s}^A}^2 \approx \frac{A R_p^2}{R_A^2} {Q_\mathrm{s}^p}^2 \approx 0.3 A^{1/3} {Q_\mathrm{s}^p}^2$$

with $R_p = 0.6\mathrm{fm}$ and $R_A = 1.1 A^{1/3}\mathrm{fm}$

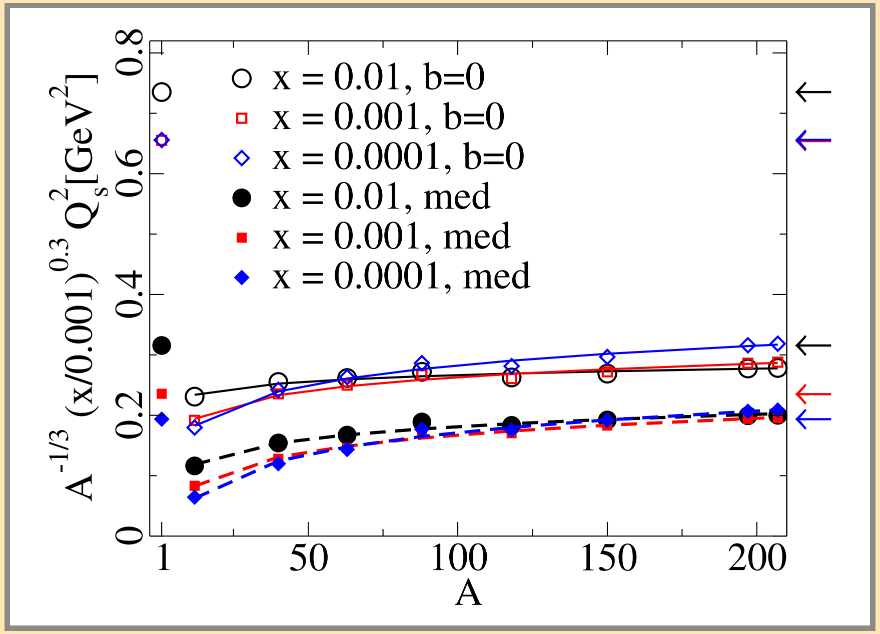




A-dependence of Q_s: in IPsat model[4]

$\sim cA^{1/3}$ and increase from DGLAP

Result
$Q_s(b_{\mathbf{med}}) \approx 1.2\mathbf{GeV}$
at RHIC,
▼
perfect agreement with CYM calculations of Glasma initial state $Q_s(b_{\mathbf{med}}) \approx 1.9\mathbf{GeV}$ at LHC. (adj)



[4] H. Kowalski, T. Lappi and R. Venugopalan, *Phys. Rev. Lett.* **100** (2008) 022303 [0705.3047].

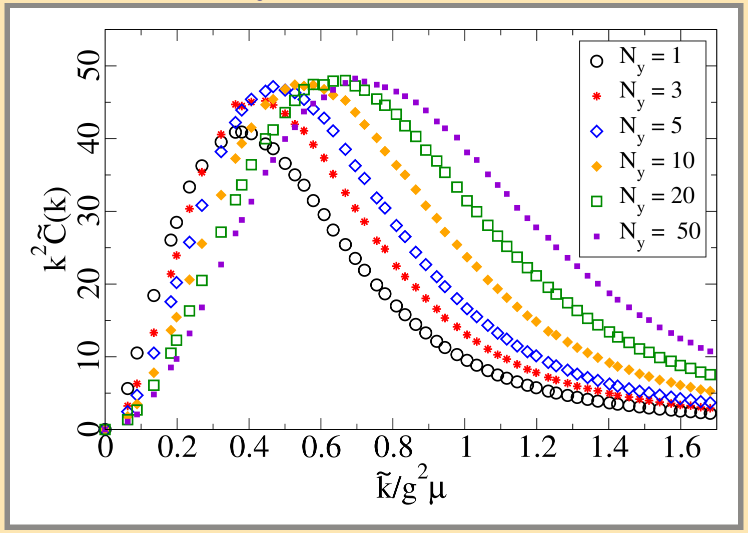
Relating DIS to AA in the MV model, Q_s vs. $g^2\mu$

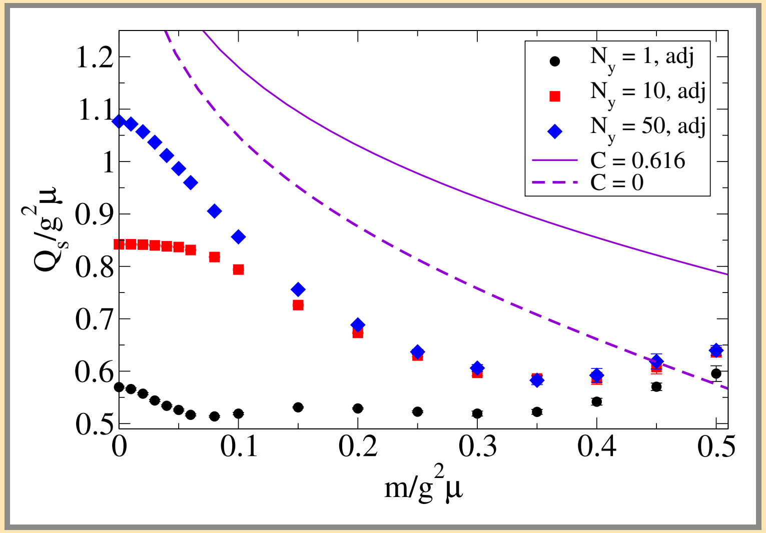
MV model, CYM has $g^2\mu$, DIS has Q_s, what is the relation[8]?

Compute the Wilson line correlator ▶ correlation length $\sim 1/Q_s$ vs. $g^2\mu$.

Result, consistently with numerical CYM calculations, is $Q_s \approx 0.6 g^2\mu$.
▶ leads to liberation coefficient $c_{\text{CYM}} \approx 1.1$.

[Numerics $N_y = 1$; Analytical results $N_y \to \infty$.] See also K. Fukushima [9]





[8] T. Lappi, *Eur. Phys. J.* **C55** (2008) 285 [0711.3039].

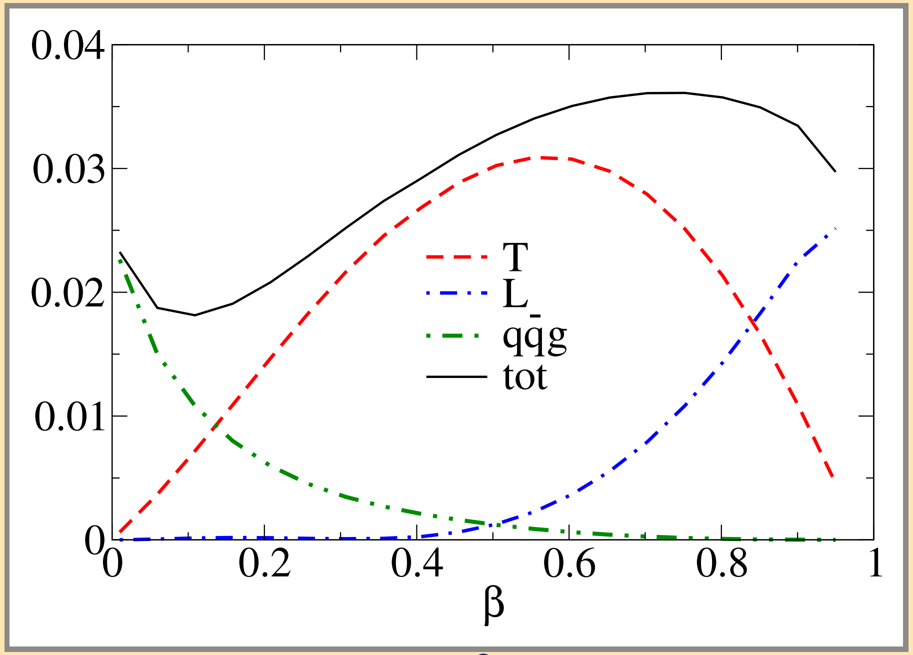
[9] K. Fukushima, *Phys. Rev.* **D77** (2008) 074005 [0711.2364].

Diffractive structure function

$$F_2^{\mathrm{D}}(x_{\mathbb{P}}, \beta, Q^2)$$

Essential regimes:

- Small $\beta \ll 1$: dominated by higher Fock ($q\bar{q}g$ etc.)
- Medium $\beta \sim 0.5$: dominated by transverse $q\bar{q}$
- Large $\beta \to 1$: longitudinal $q\bar{q}$.



Proton, $Q^2 = 5\mathrm{GeV}^2$, $x_{\mathbb{P}} = 10^{-3}$

Computing F_2^{D}

The structure function is

$$x_{\mathbb{P}} F_2^{\mathrm{D}} = \underbrace{x_{\mathbb{P}} F_{T,q\bar{q}}^{\mathrm{D}} + x_{\mathbb{P}} F_{L,q\bar{q}}^{\mathrm{D}}}_{\alpha_{\mathrm{s}}^0} + \underbrace{x_{\mathbb{P}} F_{T,q\bar{q}g}^{\mathrm{D}}}_{\alpha_{\mathrm{s}}} + \underbrace{\text{higher Fock states}}_{\alpha_{\mathrm{s}}^2}$$

$$x_{\mathbb{P}} F_{T,q\bar{q}}^{\mathrm{D}}(x_{\mathbb{P}}, \beta, Q^2) = \frac{N_{\mathrm{c}} Q^4}{16\pi^3 \beta} \sum_f e_f^2 \int_{z_0}^{1/2} \mathrm{d}z z(1-z) \left[\varepsilon^2 (z^2 + (1-z)^2)\Phi_1 + m_f^2 \Phi_0\right]$$

$$x_{\mathbb{P}} F_{L,q\bar{q}}^{\mathrm{D}}(x_{\mathbb{P}}, \beta, Q^2) = \frac{N_{\mathrm{c}} Q^6}{4\pi^3 \beta} \sum_f e_f^2 \int_{z_0}^{1/2} \mathrm{d}z z^3 (1-z)^3 \Phi_0$$

$$\Phi_n = \int \mathrm{d}^2\mathbf{b}_T \left[\int_0^\infty \mathrm{d}r r K_n(\varepsilon r) J_n(kr) \frac{\mathrm{d}\sigma_{\mathrm{dip}}}{\mathrm{d}^2\mathbf{b}_T}(\mathbf{b}_T, \mathbf{r}_T, x_{\mathbb{P}})\right]^2,$$

Integrated over t from $-\infty$ to 0, for “easy” evaluation integrating over $\mathbf{b}_T$.

Incoherent case: “Glauber MC” method, $\int \mathrm{d}^2\mathbf{b}_T$ really 2-dimensional.

$q\bar{q}g$

For $q\bar{q}g$ component we know two limits, large Q^2, finite β ("GBW") and and $\beta \to 0$, large N_c, ("MS"), combine [10]

$$x_{\mathbb{P}} F_2^{\mathbf{D}}(x_{\mathbb{P}}, \beta, Q^2) = \frac{x_{\mathbb{P}} F_{T,q\bar{q}g}^{\mathbf{D}\ \mathbf{(GBW)}}(x_{\mathbb{P}}, \beta, Q^2) \times x_{\mathbb{P}} F_{T,q\bar{q}g}^{\mathbf{D}\ \mathbf{(MS)}}(x_{\mathbb{P}}, Q^2)}{x_{\mathbb{P}} F_{T,q\bar{q}g}^{\mathbf{D}\ \mathbf{(GBW)}}(x_{\mathbb{P}}, \beta = 0, Q^2)}.$$

$$x_{\mathbb{P}} F_{T,q\bar{q}g}^{\mathrm{D}\ (\mathrm{GBW})} = \frac{\alpha_s \beta}{8\pi^4} \sum_f e_f^2 \int \mathrm{d}^2\mathbf{b}_T \int_\beta^1 \mathrm{d}z \left[\left(1 - \frac{\beta}{z}\right)^2 + \left(\frac{\beta}{z}\right)^2\right] \int_0^{Q^2} \mathrm{d}k^2 k^4 \ln\frac{Q^2}{k^2}$$

$$\times \left[\int_0^\infty \mathrm{d}r\, r K_2(\sqrt{z} k r) J_2(\sqrt{1-z} k r) \frac{\mathrm{d}\tilde{\sigma}_{\mathrm{dip}}}{\mathrm{d}^2\mathbf{b}_T}(\mathbf{b}_T, \mathbf{r}_T, x_{\mathbb{P}})\right]^2.$$

$$x_{\mathbb{P}} F_{T,q\bar{q}g}^{\mathrm{D}\ (\mathrm{MS})} = \frac{C_F \alpha_s Q^2}{4\pi^4 \alpha_{\mathrm{em}}} \int \mathrm{d}^2\mathbf{b}_T\, \mathrm{d}^2\mathbf{r}_T\, \mathrm{d}^2\mathbf{r}_T' \int_0^1 \mathrm{d}z \left|\Psi_T^{\gamma^*}(r, Q, z)\right|^2 \frac{{\mathbf{r}_T}^2}{(\mathbf{r}_T')^2 (\mathbf{r}_T - \mathbf{r}_T')^2}$$

$$\left[\mathcal{N}(\mathbf{b}_T, \mathbf{r}_T', x_{\mathbb{P}}) + \mathcal{N}(\mathbf{r}_T - \mathbf{r}_T') - \mathcal{N}(\mathbf{r}_T) - \mathcal{N}(\mathbf{r}_T')\mathcal{N}(\mathbf{r}_T - \mathbf{r}_T')\right]^2$$

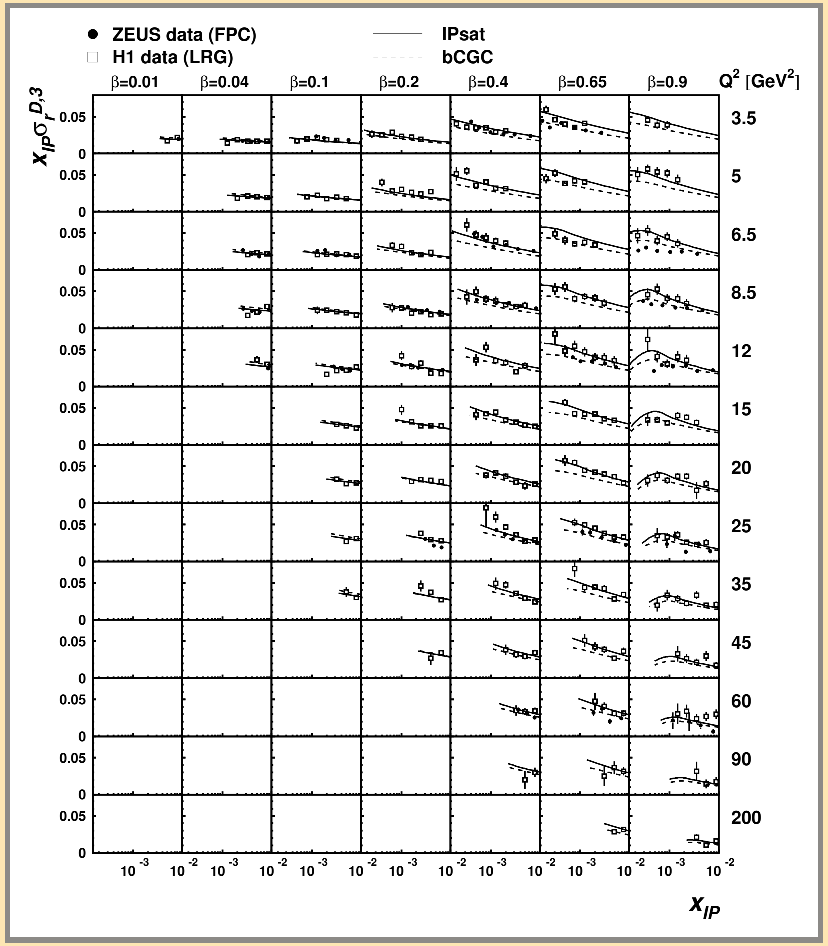
[10] C. Marquet, *Phys. Rev.* **D76** (2007) 094017 [0706.2682].

First: HERA data

Agree with HERA ep data ($x_{\mathbb{P}} < 0.01$) with $\chi^2 \sim 1$.
Caveat: must choose small $\alpha_s = 0.14$: coefficient of the $q\bar{q}g$-term.
▶ use same value for nuclei.

Note: we are plotting mostly ratios anyway, so this often cancels.

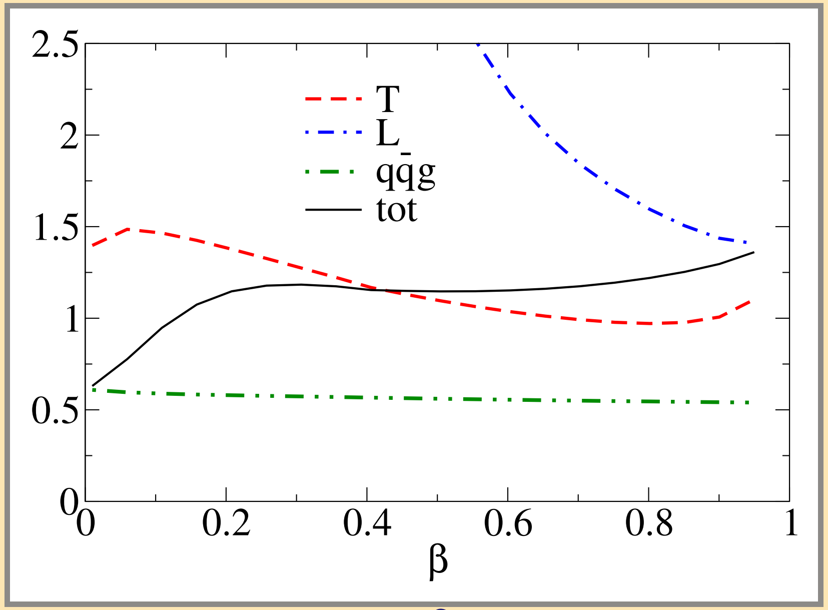
Larger $q\bar{q}g$: natural result of b-dependent form: more on this later.



Results: β-dependence

Essential regimes:

- Small $\beta \ll 1$: $q\bar{q}g$ strongly suppressed (black disk limit)
- Medium $\beta \sim 0.5$: transverse $q\bar{q}$ enhanced.
- Large $\beta \to 1$: longitudinal $q\bar{q}$ very much enhanced.



Au at $Q^2 = 5\mathrm{GeV}^2$ and $x_{\mathbb{P}} = 10^{-3}$

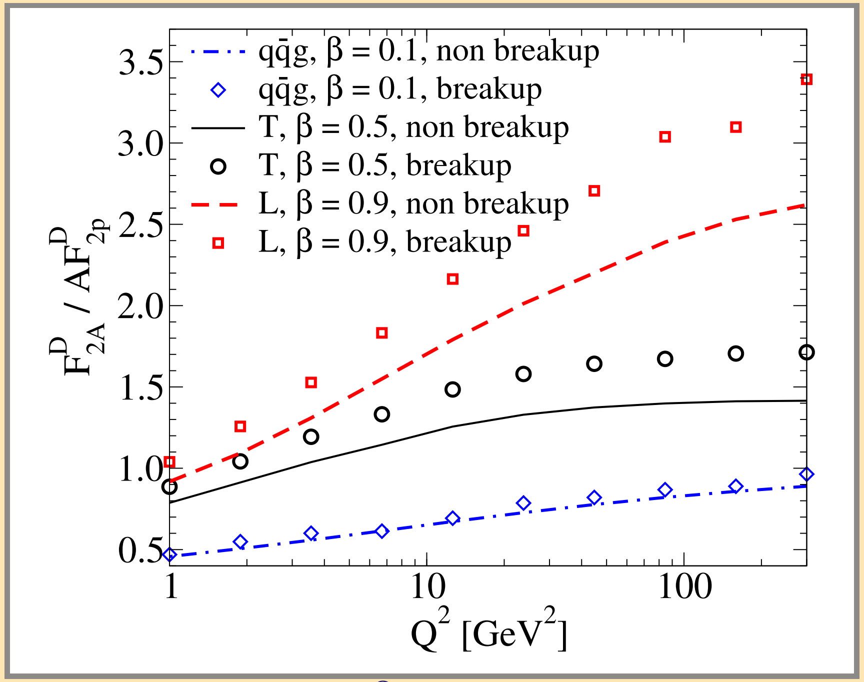
Note: plotting $\frac{F^{\mathrm{D}}_{2A,(x)}}{AF^{\mathrm{D}}_{2\ ,(x)}}$ with $x = L, T, q\bar{q}g,\ \mathrm{tot}.$

Results: Q^2-dependence

Nuclei have smaller Q^2/Q_s^2 at same $x_{\mathbb{P}}, Q^2$

▼

Nuclear enhancement of F_2^D's grows with Q^2



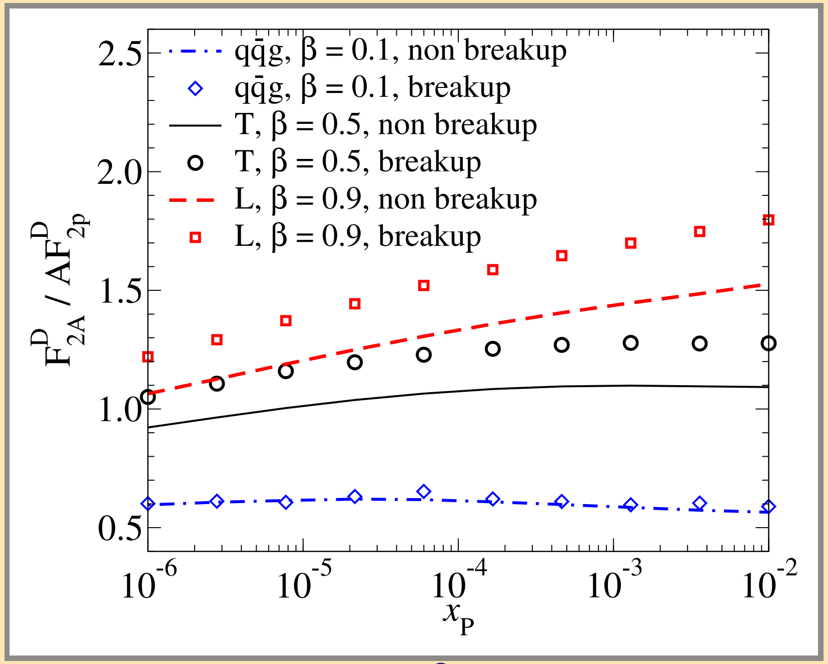
Au at $x_{\mathbb{P}} = 10^{-3}$

Non breakup = coherent
Breakup = coherent + incoherent

Results: $x_{\mathbb{P}}$-dependence

$x_{\mathbb{P}}$-dependence

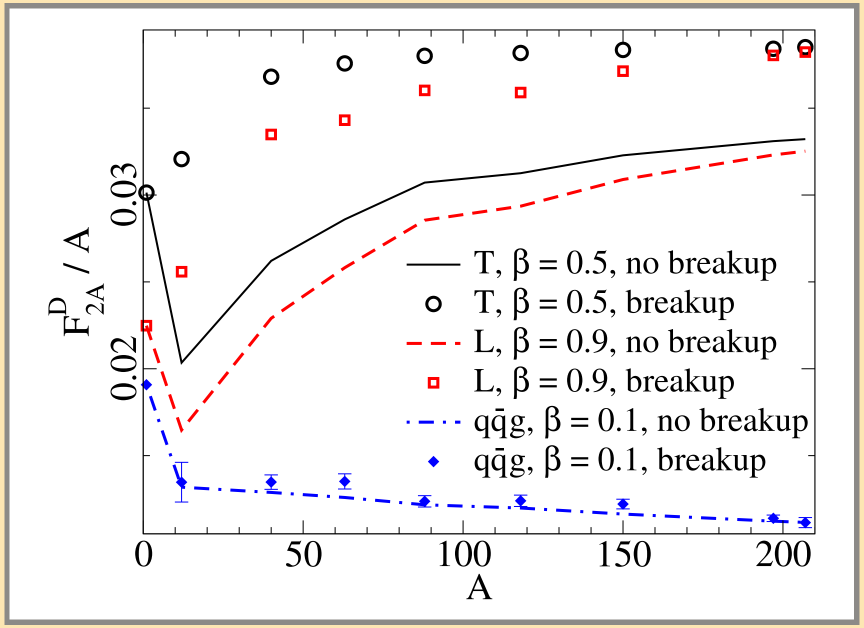
At smaller $x_{\mathbb{P}}$ also proton closer to black disk limit ▶ smaller enhancement in $q\bar{q}$-components.



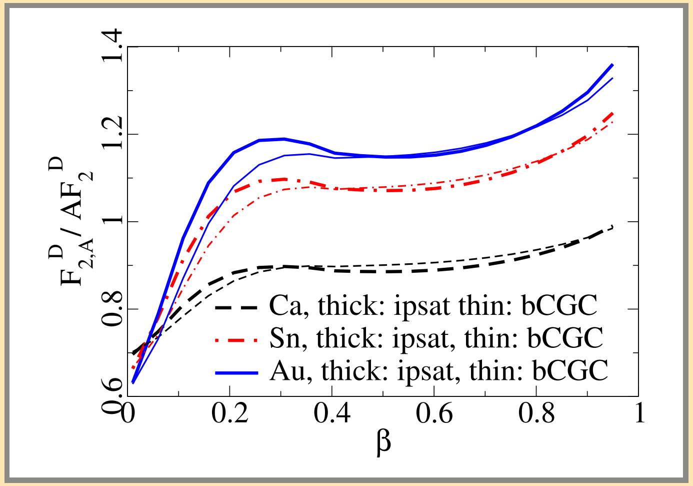
Au at $Q^2 = 5\mathrm{GeV}^2$

Results: A-dependence

Small nuclei are more dilute than proton: coherent diffraction suppressed



Different components at $x_{\mathbb{P}} = 10^{-3}$ and $x_{\mathbb{P}} = 10^{-3}$

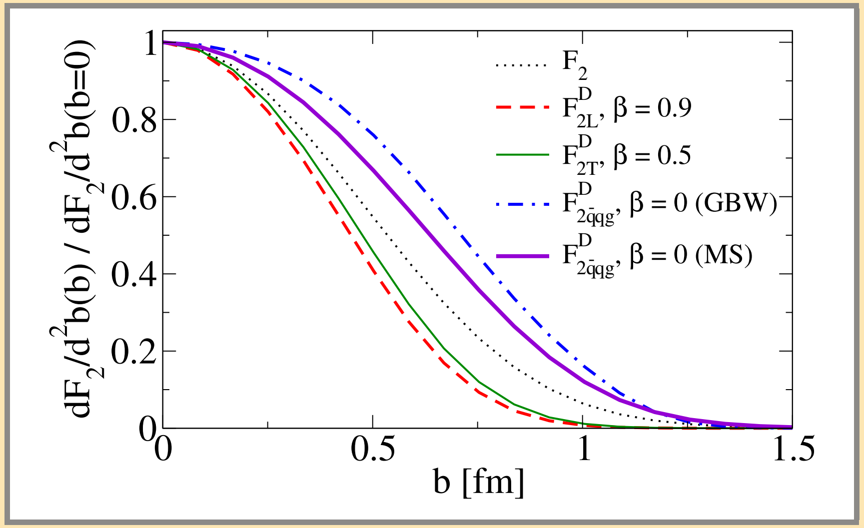


Different nuclei vs β

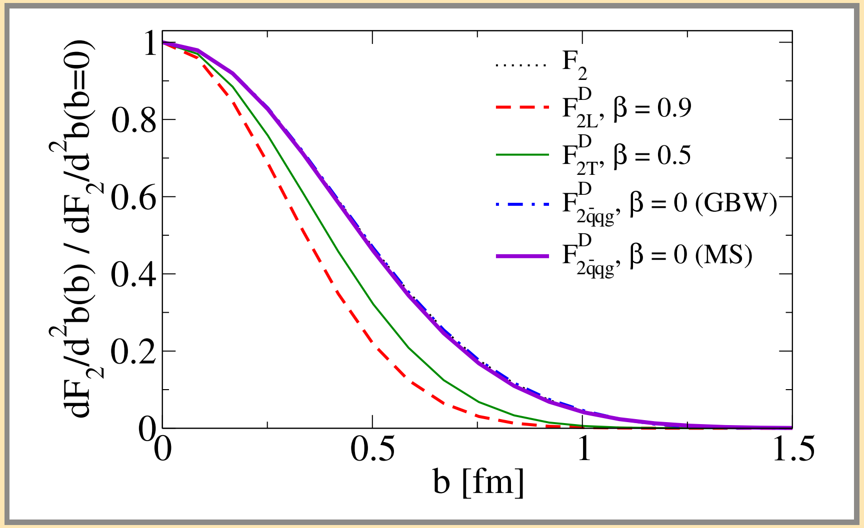
Note: impact parameter dependence of different components

Dominant impact parameters different:

$$b^{\mathrm{diff}}(q\bar{q}) < b^{\mathrm{incl}} < b^{\mathrm{diff}}(q\bar{q}g)$$



Different components at $x_{\mathbb{P}} = 10^{-3}$ and $x_{\mathbb{P}} = 10^{-3}$ $Q^2 = 1\mathbf{GeV}^2$



Different components at $x_{\mathbb{P}} = 10^{-3}$ and $x_{\mathbb{P}} = 10^{-3}$ $Q^2 = 100\mathbf{GeV}^2$

b-dependence beyond Glauber

Denote $\frac{\mathrm{d}\sigma_{\mathrm{dip}}}{\mathrm{d}^2\mathbf{b}_T}(\mathbf{r}_T, \mathbf{b}_T) = 2(1 - S(\mathbf{r}_T, \mathbf{b}_T))$.

Glauber independent scatterings: $S_A(\mathbf{r}_T, \mathbf{b}_T) = \prod_{i=1}^{A} S_p(\mathbf{r}_T, \mathbf{b}_T - \mathbf{b}_{Ti})$,

▶ for IPsat equivalent to $Q_\mathrm{s}^2(\mathbf{b}_T) \sim T_p(\mathbf{b}_T) \to \sum_{i=1}^{A} T_p(\mathbf{b}_T - \mathbf{b}_{Ti})$,

because $S_A(\mathbf{r}_T, \mathbf{b}_T) \sim e^{-{\mathbf{r}_T}^2 Q_\mathrm{s}^2/4}$

$$A^{2/3}\left(1 - e^{-{\mathbf{r}_T}^2 Q_\mathrm{s}^{p\,2} A^{1/3}/4}\right) \sim A \text{ for large } Q^2$$

Not so with BK etc. evolution, when $S_A(\mathbf{r}_T, \mathbf{b}_T) \sim e^{-\left({\mathbf{r}_T}^2 Q_\mathrm{s}^2\right)^\gamma/4}$

$$A^{2/3}\left(1 - e^{-\left({\mathbf{r}_T}^2 Q_\mathrm{s}^{p\,2} A^{1/3}\right)^\gamma/4}\right) \sim A^{2/3+\gamma/3}$$

To combine this with realistic nuclear geometry would ideally need b-dependent evolution ▶ run into confinement problems.

Good parametrization in stead of solving b-dependent evolution?

Conclusions

- Realistic b-dependence essential
- Consistent values for AA (glasma) and DIS data
- Computed diffractive structure functions F_2^{D} for nuclei

Future projects:

- Combine to a realistic b-dependent BK/JIMWLK-computation
- Global fit with AA, eA, pA, . . .

Many other works not mentioned here: Frankfurt, Strikman et. al; Nikolaev, Zakharov; Levin, Lublinsky et. al.; Kugeratski, Goncalves, Navarra . . .

Backups

Additional increase from DGLAP

"Small constant times $A^{1/3}$" gives most of KT A-dependence, but there is an additional effect: the Au to Ca – ratio is $\lesssim 3$, more than $(197/40)^{1/3} \approx 1.7$

Previous estimate: neglected increase of $xg(x, C/\mathbf{r}_T{}^2 + \mu_0^2)$ with increasing $1/\mathbf{r}_T{}^2$.

Take nuclei A and B. Saturation condition

$$\frac{\overbrace{AT_A(\mathbf{b}_T)}^{\sim A^{1/3}}\, xg(x, Q_\mathrm{s}^{A\,2})}{(Q_\mathrm{s}^A)^2} = \frac{\overbrace{BT_B(\mathbf{b}_T)}^{\sim B^{1/3}}\, xg(x, Q_\mathrm{s}^{B\,2})}{(Q_\mathrm{s}^B)^2}$$

$$\frac{Q_\mathrm{s}^{A\,2}}{Q_\mathrm{s}^{B\,2}} \sim \left(\frac{A}{B}\right)^{1/3} \times \frac{xg(x, Q_\mathrm{s}^{A\,2})}{xg(x, Q_\mathrm{s}^{B\,2})}$$

Gold/Calcium: $2.6 \approx 1.7 \times 1.5$ (around $x = 10^{-4}$?)
Bigger effect for smaller x.