Analytical expressions of the dispersive contributions of the nuclear optical model

J.M. Quesada,* A. Molina, and M. Lozano
Departamento de Física Atómica, Molecular y Nuclear,
Universidad de Sevilla, Ap. 1065, E-41080 Sevilla, Spain

R. Capote†
Departamento de Física Aplicada, Universidad de Huelva, E-21071 Huelva, Spain

J. Raynal
4 rue du Bief, 91380 Chilly-Mazarin, France
(Dated: June 6, 2003)

Analytical solutions of dispersion relations in the nuclear optical model have been found for both imaginary volume and surface potentials. A standard Brown-Rho shape has been assumed for the volume imaginary term and a Brown-Rho shape multiplied by a decreasing exponential for the surface contribution. The analytical solutions are valid for any even value of the exponent appearing in these functional forms.

PACS numbers: 11.55.Fv, 24.10.Ht

A significant contribution to the optical model theory during the last two decades can be considered the work of Mahaux and co-workers on dispersive optical model analysis[1–5]. The unified description of nuclear mean field in dispersive optical model is accomplished by using a dispersion relation, which links the real and absorptive terms of the optical model potential (OMP). The additional constraint imposed by dispersion relation helps to reduce the ambiguities in deriving phenomenological OMP parameters from the experimental data. In a dispersion relation treatment, the real central potential strength consists of a term which varies slowly with energy E, the so called Hartree-Fock (HF) term, \( V_{HF}(r, E) \), plus a correction term, \( \Delta V(r, E) \), which is calculated using a dispersion relation. The depth of the dispersive term of the potential \( \Delta V(r, E) \) can be written as

\[
\Delta V(r, E) = \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{W(r, E')}{(E' - E)^2} dE'
\]

(1)

Being \( W(r, E) \) the imaginary part of the OMP. Assuming that \( W(r, E = E_F) = 0 \) and \( \Delta V(r, E = E_F) = 0 \), where \( E_F \) is the Fermi energy, then equation (1) can be written in the substracted form:

\[
\Delta V(r, E) = \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{W(r, E')}{(E' - E)^2} dE'
\]

(2)

With the assumption that \( W(r, E) \) be symmetric respect to the Fermi energy, the equation (2) can be expressed in a form which is stable under numerical treatment [6], namely:

\[
\Delta V(r, E) = \frac{2}{\pi} (E - E_F) \int_{E_F}^{\infty} \frac{W(r, E') - W(r, E)}{(E' - E)^2 - (E - E_F)^2} dE'
\]

(3)

The dispersive term \( \Delta V(r, E) \) is divided into two terms \( \Delta V_{HF}(r, E) \) and \( \Delta V_S(r, E) \), which arise through dispersion relations (1) from the volume \( W_{HF}(r, E) \) and surface \( W_S(r, E) \) imaginary potentials respectively. If the imaginary potential geometry is energy dependent, the radial dependence of the dispersive correction cannot be expressed using a Woods-Saxon form factor \( f_{ws}(r) \). However, to simplify the problem, the OMP geometry parameters are usually assumed to be energy independent. In this case, \( \Delta V(r, E) = \Delta V(E) f_{ws}(r) \) and the energy dependence of the real volume \( V_{HF}(E) \) and surface \( V_S(E) \) parts of the dispersive OMP are given by:

\[
V_{HF}(E) = V_{HF}(E) + \Delta V_{HF}(E)
\]

\[
V_S(E) = V_S(E) + \Delta V_S(E)
\]

(4)

It is useful to represent the variation of surface \( W_{HF}(E) \) and volume \( W_{HF}(E) \) absorption potential depths with energy in functional forms suitable for the dispersive optical model analysis. An energy dependence for the imaginary volume term has been suggested in studies of nuclear matter theory [7]:

\[
W_{HF}(E) = A_{HF} \frac{(E - E_F)^n}{(E - E_F)^m + (B_{HF})^m}
\]

(5)

where \( A_{HF} \) and \( B_{HF} \) are undetermined constants. Brown and Rho [7], propose \( n = 2 \), while Mahaux and Sartor [2] have suggested \( n = 4 \). An energy dependence for the imaginary-surface term has been suggested by Delaroche et al [6] to be:

\[
W_S(E) = A_S \frac{(E - E_F)^m}{(E - E_F)^m + (B_S)^m} \exp(-C_S |E - E_F|)
\]

(6)

*quesada@us.es
†Permanent address: Centro de Estudios Aplicados al Desarrollo Nuclear, Ap. 100, Miramar, La Habana, Cuba
where $m = 2, 4$ and $A_s, B_s$ and $C_s$ are undetermined constants.

According to equations (5) and (6) the imaginary part of the OMP is assumed to be zero at $E = E_p$ and nonzero everywhere else. A more realistic parameterization of $W_{\nu}(E)$ and $W_s(E)$ forces these terms to be zero in some region around the Fermi energy. A physically reasonable energy for defining such a region is the average energy of the single-particle states $E_p$ [4].

Therefore a new definition for imaginary part of the OMP can be written as:

$$
W_{\nu}(E) = \begin{cases} 
0 & E < E_p \\
A_{\nu} \frac{(E - E_p)^n}{(E - E_p)^m + (B_s)^n} & E \geq E_p
\end{cases}
$$

and likewise for surface absorption.

The integrand can be replaced by its expressions in terms of poles and residues:

$$
\frac{E_{\nu}}{\pi} \frac{U^m}{(U + E_+)(U + E_+)} = \frac{1}{\pi} \left\{ \sum_{j=1}^{m} \frac{\text{Res}(p_j)}{U - p_j} + \frac{\text{Res}(E_+)}{U + E_+} + \frac{\text{Res}(E_0)}{U + E_0} \right\}
$$

where the $p_j$ are the $m$ zeroes of $(U^m + B^m)$ and the Res$(p_j)$ their residue, that is:

$$
p_j = B \exp \left( \frac{2j - 1}{m} \pi \right)
$$

$$
\text{Res}(p_j) = \frac{E_{\nu}}{m} \frac{p_j}{(p_j + E_+)(p_j + E_0)}
$$

where $\pm E_+$ and $-E_0$ are the poles of $U \mp E_+$ and $U + E_0$, whereas Res$(\pm E_+)$ and Res$(-E_0)$ are their residues respectively.

As was pointed out by Raynal [10] the contribution of each complex pole $p_j$ to the surface dispersive integral is:

$$
\int_0^\infty \text{Res}(p_j) e^{-CU} dU = \int_{-cp_j}^{\infty} \text{Exp}(-z) dz \equiv \text{Res}(p_j) e^{-cp_j} E_1(-cp_j)
$$

where $E_1(z)$ is the Exponential Integral Function $E_1$ [11]. For the real poles corresponding to the second term in the

$$
\Delta V_s(E) = \frac{E_{\nu}}{\pi} \int_0^\infty \frac{W_s(E')}{E' - E}(E' - E_p) dE' = \frac{E_{\nu}}{\pi} \int_0^\infty \frac{U^m \exp(-CU)}{(U + E_-)(U + E_+ - E_p)} dU + \frac{E_{\nu}}{\pi} \int_0^\infty \frac{U^m \exp(-CU)}{(U + E_-)(U + E_+)} dU
$$

and the same expression without exponential for the volume potential $W_{\nu}(E)$, given by equations (7) and (9).
right side of (11), the contribution of each one is given by:

\[
\int_0^\infty \frac{\text{Res}(\mp E_{\pm}) e^{-C E_{\pm}}}{U \mp E_{\pm}} dU =
\]

\[
= \text{Res}(\mp E_{\pm}) e^{+CE_{\pm}} \frac{\text{P}}{x} \int_{-CE_{\pm}}^\infty \frac{\exp(-x)}{x} dx \equiv
\]

\[
\equiv -\text{Res}(\mp E_{\pm}) e^{+CE_{\pm}} \text{Ei}(\pm CE_{\pm})\]  

(17)

where \( \text{Ei}(x) \) is the \textit{Exponential Integral Function} \( \text{Ei} \) \[11\].

The contributions coming from the third side of the right side of eq. (11) in the integral (10) cancel.

For the volume dispersive integral the exponential is missed in the integrand, therefore each pole gives a divergent contribution. Nevertheless their sum is a finite quantity, which can be calculated by taking the proper limit.

We are quoting below exact expressions for the surface and volume dispersive integrals for any even values of \( m \) and \( n \) in the surface and volume imaginary potentials respectively. This make a difference to ref. [8], where, in surface dispersive integral with \( m = 4 \), approximations were made limiting the energy range of validity for given analytical expressions.

For the case of the dispersion relation using surface imaginary potential \( W_s(E) \) with \( m \) even according to equations (8) and (9), the dispersive correction is

\[
\Delta V_s(E) = \frac{A}{\pi} \left\{ \sum_{j=1}^{n} Z_j e^{-p_j C} E_1(-p_j C) - \text{Res}(-E_+) e^{CE_+} \text{Ei}(-CE_+) \right\}
\]

(18)

where \( Z_j \) comes from the sum of residues \( \text{Res}(p_j) \) in the two integrals (10) and is given by

\[
Z_j = \frac{E_x}{m} \frac{1}{(p_j + E_n)(p_j + E_+)(p_j - E_-)}
\]

(19)

For the case of the dispersion relation using the volume imaginary potential \( W_v(E) \) with \( n \) even according to equations (7) and (9), the dispersive correction is

\[
\Delta V_v(E) = -\frac{A}{\pi} \left\{ \sum_{j=1}^{n} Z_j \ln(-p_j) + \text{Res}(-E_+) \ln E_+ + \text{Res}(-E_-) \ln|E_-| \right\}
\]

(20)

In conclusion, we have developed analytical solutions of dispersion relations for the volume and surface terms of the OMP. The formulæ are compact and easy to implement in current generation of the optical model parameter search codes. We stress that the solutions are valid for any even exponent \( n \) or \( m \).

This work was supported by Junta de Andalucía and the Spanish CICYT under Contracts PB1998-1111, FPA2001-0144-C05-03 and FPA2001-4960-E and by the European Union under Contract FKIW-CT-2000-00107.