Emergences in Quantum Measurement Processes

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Abstract
Quantum mechanics is acknowledged as the fundamental theory on which the whole fabric of physics is supposed to rely. And yet, the features of quantum measurements, processes that provide information about microscopic objects, seem to contradict the principles of quantum mechanics. We make a qualitative presentation of this long standing problem and give an idea of recent progress in the elucidation of the paradox. Although governed solely by the quantum equations of motion, the dynamical process involving the tested system and the measuring apparatus veils the quantum oddities that oppose our standard logic and gives rise to the expected properties of measurements. In spite of the irreducibly probabilistic nature of the underlying quantum physics, classical concepts emerge, such as standard probabilities, ordinary correlations, disappearance of quantum fluctuations, and the possibility of making statements about individual systems.

Keywords
quantum physics, probability, measurement, emergence, dynamics, information, correlation, uniqueness, irreversibility

Quantum mechanics has provided the most precise results obtained in science in the field of atomic physics and quantum optics, allowing the realization of atomic clocks with an accuracy of a second over a hundred million years. Moreover, physics at the macroscopic scale as well as chemistry, and hence the sciences that derive there from, also rely on quantum theory. Of course, new properties, most often unexpected, appear when more and more complex objects are considered, but one expects that no principles other than those that govern the elementary constituents are needed to explain them. It is therefore a challenge for theorists to understand the emergence of macroscopic concepts from quantum physics. While this task has been achieved for most branches of physics, the long standing problem of quantum measurements remains controversial. Their properties seem at variance with the fundamental
quantum laws, and many proposals have been set forth to solve this problem which has deep implications for the very interpretation of the quantum laws. We shall try to indicate below where the difficulty lies and to give an idea of a recent proposal that reconciles the features of quantum measurements with the current quantum theory.

1. The Principles of Quantum Mechanics

Strangely, although quantum mechanics is our most fundamental theory, it involves some fuzziness in contrast with the determinist conceptions about macroscopic physics that we inherited from the nineteenth century. Indeed, it is irreducibly probabilistic. Probabilities are usually introduced because we deal with physical quantities supposed to take values which are perfectly defined but on which information is missing. For instance, nothing in classical physics prevents us from imagining that the position $x$ and the velocity $v$ of a particle take some well-defined values; if randomness is present it is only because we do not know their exact values. In quantum physics, we have to abandon this intuitive representation of physical quantities by ordinary numbers with well-defined values. Probabilities, statistical fluctuations and randomness are unavoidable: they arise from the very nature of the microscopic physical quantities. In fact, the whole development of science during the last century has compelled us to represent the physical quantities pertaining to some system, such as the position $x$ or the velocity $v$ of a particle of mass $m$, by elements of a non-commutative algebra. These mathematical objects, referred to as "observables", behave as random variables (in that they have expectation values, variances, correlations, and higher order moments) except that the product of two observables depends on their order. Whereas in classical physics, quantities like $x$ and $v$ commute, that is, the products $xv$ and $vx$ are identical, we have in quantum physics to distinguish these two products. The difference, in this case $xv - vx = i\hbar / m$ (with $i^2 = -1$), is provided by the rules of quantum mechanics in terms of Planck's constant $\hbar$. Many odd quantum properties that shock our daily experience and even our logics are rooted in the non-commutability of physical observables.

In a given physical situation, one can associate with any observable $A$ of the considered system its expectation value $\langle A \rangle$. From the expectation value of $A^2$ we get the statistical fluctuation $\Delta A$ of $A$ defined by $(\Delta A)^2 = \langle (A - \langle A \rangle)^2 \rangle$. In general, this quantity does not vanish so that $A$ does not have a well-defined value. For instance, the non-commutation of the position $x$ and the velocity $v$
of a particle entails Heisenberg’s inequality $\Delta x \cdot \Delta v \geq \hbar/2m$, which sets a lower bound on the fluctuations $\Delta x$ and $\Delta v$. The uncertainty implied by this inequality is not simply a result of our ignorance, but indicates the incompatibility of these quantities: we cannot even imagine that they simultaneously take well-defined values.

More generally the state of a physical system at a given time is characterized by a mathematical object called the “density operator” $D$, which allows us to evaluate the expectation values of the whole set of observables of this system. Such expectation values include statistical fluctuations and correlations. Wave functions correspond to special cases of density operators; termed “pure states”, they carry the largest possible amount of information but are still probabilistic. Density operators are more general, as they encompass two kinds of indeterminations, the quantum ones present in pure states, and those due to a further lack of information; however these two types of indeterminations cannot be disentangled unambiguously. The representation of a state by a density operator is probabilistic and blurred, but nevertheless complete in the sense that it informs us about all the observables. (Most modern interpretations of quantum mechanics exclude the existence of “hidden variables” that would take well-defined values but would be ill-known.) Density operators thus characterize our information and not the system per se. They play the role of probability distributions, but differ from standard probabilities due to the non-commutative nature of the physical quantities; their strange behaviour is shown by the Heisenberg inequalities for fluctuations, or by the violation of the Bell inequalities for correlations.\footnote{John Stewart Bell has shown the existence of inequalities satisfied by the pair-wise correlations between a set of four random variables governed by ordinary probability theory. If physical quantities were represented by such classical variables, these inequalities ought to be satisfied in any experiment. Quantum theory predicts that correlations among observables may violate them in some circumstances, and this prediction has been verified experimentally.}

The principles of quantum mechanics involve in addition the Liouville – von Neumann equation of motion, which generalizes the Schrödinger equation and governs the evolution of the density operator of an isolated system. The statistical interpretation of quantum mechanics relies on the sole principles that we have just stated.
2. Ideal Quantum Measurements

Quantum measurements are processes that allow us to draw statistical information about the observables of a system. We focus on ideal measurements, extensively analyzed by theoreticians (though seldom performed by experimentalists). The difficult problems that they raise should be elucidated for a complete understanding of the quantum principles. Indeed, the laws of measurements, stated in the early days of quantum mechanics and recalled below, seem foreign to the principles of the fundamental theory.

Consider a system $S$ on which some observable $s$ is measured. During the measurement process, $S$ interacts with an apparatus $A$. This apparatus is modified by the process, as it contains a pointer providing us indirectly with information about the microscopic (random) observable $s$. The probabilistic nature of quantum mechanics implies that the outcome is most often not uniquely predictable; the possible values $s_i$ of $s$ that may be obtained are determined by the algebraic structure of the observable set. In fact, we observe or register at the end of the experiment an indication $A_i$ of the pointer, and we rely on the correlations between $S$ and $A$ produced by the process to infer from $A_i$ the corresponding value $s_i$ of $s$.

For a given observable $s$, different outcomes $i$ for both the system and the pointer are thus allowed, and each one arises with some (ordinary) probability $p_i$. This probability $p_i$ is expressed in terms of the initial density operator of the system $S$ by Born’s rule of quantum mechanics, adapted to the density operator formalism.

In a classical measurement, nothing prevents us from imagining that $S$ remains practically unchanged throughout the process while the outcome is registered by $A$. In contrast, a quantum measurement must in general perturb $S$. This perturbation is minimal for ideal measurements, and is termed von Neumann’s reduction. We characterize it as follows. Consider the set of observables of the compound system $S+A$. It contains on the one hand $s$ and the observables that commute with $s$, on the other hand a complementary set of observables incompatible with $s$, i.e., which do not commute with $s$. The initial density operator $D(0)$ of $S+A$ gathers the expectation values of both sets. In an ideal measurement the whole information contained in $D(0)$ about $s$ and the observables compatible with $s$ are preserved. However, the reduction, a perturbation having no equivalent in classical physics, takes place. It consists in the elimination, during the process, of all information about the second, complementary set. Thus, destruction by the apparatus of information about the second set is a price paid for gaining information about the first set.
3. Conceptual Problems Raised by Quantum Measurements

Several features of an ideal quantum measurement seem to be at variance with the principles of quantum mechanics recalled above. The reduction is obviously an irreversible process since part of the density operator $D(0)$ is irremediably lost. However, the compound system $S+A$ is isolated, and should therefore evolve according to the Liouville – von Neumann equation, which is reversible, so that the whole information about $S+A$ contained in the initial state should be retained (though transferred from some degrees of freedom to other ones). The same difficulty arises for the registration, an irreversible creation of a permanent indication on the pointer of the apparatus.

Moreover, the equation of motion is linear, and the whole set of repeated runs of the measurement is issued from the initial state $D(0)$. Then, how is it that one may reach, from this same single initial state, several possible different final states $D_i$ for $S+A$?

The most dramatic contradiction, usually referred to as the quantum measurement problem, is the following. Being irreducibly probabilistic, quantum theory, even when it deals with pure states, describes statistical ensembles, not individual systems. As already stressed, what we call the "state of a system" is the collection of expectation values of its whole set of observables; these expectation values are not merely properties of this single system, but probabilistic quantities referring to a large set $E$ of similarly prepared systems. When we pretend to describe an individual system, we include it, explicitly or implicitly, in such a real or thought ensemble. Thus, a quantum mechanical study of the evolution of the coupled system $S+A$ during a measurement process can only account for the statistical properties of a large set of runs of repeated measurements.

Nevertheless, the properties of measurements listed above require the consideration of individual processes. For instance, Born’s rule yields the proportion $p_i$ of runs that have produced the outcome $i$. We need to explain how these runs can be distinguished from the others, so as to assign to their outcome the density operator $D_i$. Quantum mechanics even seems unable to explain why an individual run yields some well defined outcome, since there is no intuitive interpretation of a density operator for a single event. We will discuss this point further below.

Many ideas have been set forth to solve these paradoxes including modifications of the quantum theory. The most consistent and conservative approach consists in trying to explain how the special features of measurements may emerge solely from the principles of standard quantum mechanics in spite of...
obvious qualitative and conceptual differences between these principles and the properties of measurements. The study of models is a privileged means for elucidating this question.

4. Emergence of Irreversibility

We will first rely on an analogy with a well-known problem. The irreversibility of a measurement process seems to contradict the reversibility of the Liouville – von Neumann equation of motion. Likewise, the celebrated paradox of irreversibility in statistical mechanics opposes the irreversibility expressed by the Second Law of thermodynamics to the reversibility of the microscopic equations of motion that govern the elementary constituents of matter. For instance a gas enclosed in a vessel and far from equilibrium eventually reaches a state in which its density and temperature have become uniform and local macroscopic flow has disappeared. However, at the microscopic scale, this gas is constituted of many identical molecules which undergo reversible collisions between free motions. How can such a simple microscopic structure and such a simple dynamics underlie the thermodynamic properties?

This paradox has been elucidated long ago. It has been shown that, in spite of their qualitative differences, the macroscopic properties issue from the microscopic ones. A key point is the extremely large number of molecules. Owing to this large number, approximations that change some qualitative behaviour may become nearly exact, and this allows us to understand the emergence of new properties. In particular, a cascade of correlations between larger and larger sets of molecules is created by their successive collisions. The memory of the initial inhomogeneous state is retained within such correlations while the density tends to become uniform as a consequence of the reversibility of each collision. These many-particle correlations, however, are inaccessible to any observation and can be discarded. In principle, they might give rise to recurrences, leading the gas back to its original configuration, but the large number of molecules makes the recurrence time enormous, much larger than the lifetime of the universe. Finally, probabilistic considerations are essential. In particular, an evolution that would simulate a reversal of time, the gas becoming more and more inhomogeneous, is not excluded for some special initial states, but the necessarily macroscopic preparation makes such initial conditions very unlikely.

Similar ingredients are used in the explanation of the von Neumann reduction and of the registration, two irreversible features of quantum measure-
ments. To describe the interaction process between the system S and the apparatus A in the quantum framework, we start from the initial density operator $D(0)$ that describes the compound system $S+A$ at the initial time $t=0$. We study the evolution of $D(t)$ by solving the Liouville – von Neumann equation, and this provides us with the density operator $D(t_f)$ of $S+A$ at the final time $t_f$.

In this derivation, we rely on the fact that the apparatus is macroscopic. This property allows us to justify some approximations which entail on the one hand the reduction of the state $D(t)$, on the other hand the registration of the outcome by the pointer of A. The final state is thus found in the form

$$D(t_f) = \sum_i p_i D_i.$$  

Each density operator $D_i$ alone would describe a situation of thermodynamic equilibrium in which the tested observable $s$ of S takes the value $s_i$ (without fluctuation), and the pointer takes the corresponding value $A_i$.

A part of the apparatus can behave as a pointer only if it may reach several possible stable states, macroscopically distinguishable. The outcome of the measurement can then be registered when one or another of these states is reached at the end of the process. This property also requires a large number of elementary constituents in the pointer, since systems with a small number of constituents have in general only a single equilibrium state. Different phases may exist in macroscopic pieces of matter, for instance, when some invariance is spontaneously broken; each phase is then stable because it is separated from the others by energy barriers which make transitions extremely improbable and unobservable.

As in the relaxation of the gas, owing to the large size of the apparatus, interactions transfer the initially existing controllable properties of $S+A$ towards more and more intricate observables. The resulting correlations involve such a large number of particles that they are out of reach. Thus, information flows towards inaccessible correlations and cannot be retrieved. This loss is irremediable because recurrence times are again enormous. Probability plays a major role in the derivation of the above expression of $D(t)$, as we can disregard situations that are extremely unlikely.

Finally, some conditions must be satisfied by the initial state of the apparatus to ensure a faithful measurement. It is interesting to note that, although the properties of the measurement will emerge only from the microscopic quantum laws, the initial conditions are fixed through macroscopic means.
5. Emergence of Uniqueness

Probabilities lie in the heart of quantum theory itself on which we are relying to explain the measurement process. The irreducible nature of such probabilities will bring in a new difficulty. As any quantum state, the final density operator $D(t_f)$ found by solving the Liouville – von Neumann equation does not describe the outcome of the individual runs of the measurement but only the statistics of the outcomes of a large set $E$ of runs. Once we have determined $D(t_f)$, the quantum measurement problem remains still open. We need to understand why each individual run provides a well defined indication $A_i$ of the pointer, how the set $E$ of runs may be split into subsets $E_i$ characterized by this indication $A_i$ and described by the density operators $D_i$ (which correlate $s_i$ and $A_i$), and to understand how Born’s rule relates the relative proportion $p_i$ of runs in each subset $E_i$ to the density operator $D(0)$. Such questions are subtle and will require somewhat technical explanations.

The above expression (1) of $D(t_f)$, although suggestive, is not sufficient to ensure these required properties, due to a quantum oddity. In ordinary probability theory, the knowledge of the probability distribution that describes a large statistical ensemble allows us to associate with each separate event its probability, identified to the proportion of similar events in this ensemble. The situation is different in quantum mechanics and rather subtle. We wish to describe probabilistically the final state of some individual run $Y$ of the measurement by assigning to it a density operator $D'$. In fact, $Y$ should be regarded as an element of a statistical ensemble described by $D'$. We thus consider some subset $E'$ of runs, itself embedded in $E$, and containing $Y$. The assignment of $D'$ to $E'$ must be compatible with the assignment of $D(t_f)$ to the whole set $E$ of runs. We are thus led to analyze an arbitrary splitting of $E$ into sub-ensembles $E'_k$ and to relate to one another the corresponding density operators. If we denote by $D'_k$ the state associated with $E'_k$ and by $\nu'_k$ the proportion of runs contained in $E'_k$, the fact that $E$ is the union of the subsets $E'_k$, implies that the known state $D(t_f)$ should be decomposable into the weighted sum

$$D(t_f) = \sum_k \nu'_k D'_k. \tag{2}$$

The particular run $Y$ then seems to belong to one of the subsets $E'_k$ and to be described by the corresponding state $D'_k$. However, there exist many such decompositions of $D(t_f)$, even if we consider the most detailed decompositions in which the states $D'_k$ are pure. This situation contrasts with classical statistical mechanics, where the decomposition of a density in phase space, the equiv-
alent of a density operator, in terms of points in phase space, the most detailed classical states, is unique. Let us then consider a decomposition other than (2),

\[ D(t_f) = \sum l \nu' l D'' l, \quad (3) \]

associated with a splitting of \( E \) into subsets \( E'' l \). Then, if both splittings are real, and are not purely formal manipulations, the run \( Y \) should belong to two subsets, \( E' k \) and \( E'' l \), and its properties should be accounted for by both \( D' k \) and \( D'' l \). We thus stumble over a contradiction: the two density operators \( D' k \) and \( D'' l \), supposed to describe the same run \( Y \), are in general incompatible: they yield different predictions about physical quantities. Quantum mechanics is plagued by this ambiguity of the decompositions of \( D(t_f) \). All are mathematically correct, but at most one among them may have a physical meaning and represent the outcome of an actual individual run.

We must therefore extend the analysis of quantum measurement models so as to account, not only for the whole ensemble \( E \) of runs, but also for its physical sub-ensembles. We thus consider an arbitrary density operator \( D' \) issued from some decomposition (2) of \( D(t_f) \) and associated with a subset \( E' \). Using the Liouville – von Neumann equation, we study its evolution during the last stage of the process, for models involving a sufficiently intricate interaction within the macroscopic apparatus. We also rely on the specific form of the density operator \( D' \) compatible with \( D(t_f) \), which combines the different possible final states of the pointer. The states \( D_i \) entering (1) are special cases of such states \( D' \); they are stationary. However, it turns out that all other states \( D' \) undergo a rapid relaxation towards some linear combination of the particular states \( D_i \):

\[ D' \rightarrow \sum q' i D_i. \quad (4) \]

This dynamical instability implies that the final state of any subset of runs of the measurement has in all cases the form (4). The only physically meaningful decomposition of \( D(t_f) \) is the one exhibited by (1), all other ones (2) are excluded by the dynamics.

The required properties of ideal measurements then emerge. The only states in which any individual run \( Y \) can end up are the states \( D_i \), so that both the indication \( A_i \) of the pointer and the value \( s_i \) of the observable \( s \) of \( S \) are well-defined. The observation and processing of the macroscopic value \( A_i \) allow us to split unambiguously a set of measurements into subsets characterized by this
value and described by the state $D_i$. Then the "collapse" of the state, which consists in the replacement of $D(0)$ before measurement by $D_i$ afterwards, can simply be interpreted as an updating of our knowledge embedded in the density operator, which is permitted by our gain of information. The weights $p_i$, entering (1), which are expressed in terms of the initial state $D(0)$ through the solution of the dynamical equations, are interpreted as the proportion of runs belonging to the subset $E_i$, that is, as ordinary probabilities in the sense of relative frequencies. The assignment of the state $D_i$ to this subset $E_i$ implies the existence of complete classical correlations between $s_i$ and $A_j$, without the peculiarities of quantum correlations.

6. Measurements as Dynamical Emergence of Classical Concepts

Altogether, the solution of models demonstrates how the properties of measurements, in spite of their apparent contradiction with the basic principles of quantum mechanics, emerge from these very principles without further hypotheses. Such an emergence is a consequence of the dynamics of the joint system constituted by the tested object and the apparatus; the time scales involved in the various stages of the process can be evaluated. The specific features of ideal measurements emerge owing to the many degrees of freedom of the apparatus, to suitable interactions between $S$ and $A$ and between the constituents of $A$, and to probabilistic considerations: events that would violate the expected properties of the measurement are mathematically allowed but physically irrelevant due to their very small likelihood.

Measurements provide us with information about quantum systems through observation or registration of the indications of the apparatus. In this transfer from a microscopic to a macroscopic object, many quantum properties get hidden, thus allowing classical reasoning. We lose track, at least in part, of the strange consequences of the non-commutability of the observables, the deep foundation of quantum physics. We become able to make statements about individual events. Well-defined values $s_i$ emerge in each run for the tested observable $s$, although $s$ was initially subject to statistical fluctuations. Ordinary probabilities entering Born’s rule emerge from the non-commutative probabilities embedded in the density operator. Ordinary correlations also emerge; they are created between $S$ and $A$, and allow us to infer some properties of $S$ through the processing of the indications of the pointer. Remarkably, the type of emergences we acknowledge here is more subtle than the celebrated emergence of thermodynamics from statistical mechanics, as it does not bear on phenomena, but on the very concepts of physics.
References

This essay introduces some among the ideas presented in full detail in the first reference below. More can be found in the second reference about its information theory aspects. Both texts may be downloaded from http://ipht.cea.fr (t11/166 and t04/185).

