Blackfolds

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+ ***to appear***

Why study D>4 gravity?

- Main motivation (at least for me!):
  - better understanding of gravity (i.e. of what ***spacetime*** can do)

- In General Relativity in vacuum

$$R_{\mu\nu}=0$$

  $\exists$ only one parameter for tuning: D

  - BHs exhibit novel behavior for $D > 4$

Why study D>4 gravity?

- Also, for applications to:
  - String/M-theory
  - AdS/CFT

    (+its derivatives: AdS/QGP, AdS/cond-mat etc)
  - Math
  - TeV Gravity (bh's @ colliders…)
  - etc

- When first found, black hole solutions have always been "answers waiting for a question"

4D vs hi-D Black Holes: Size matters

- Main novel feature of D>4 BHs: in some regimes they're characterized by two widely separate scales:

$$\ell_M \sim (GM)^{1/(D-3)}\,, \qquad \ell_J \sim J/M$$

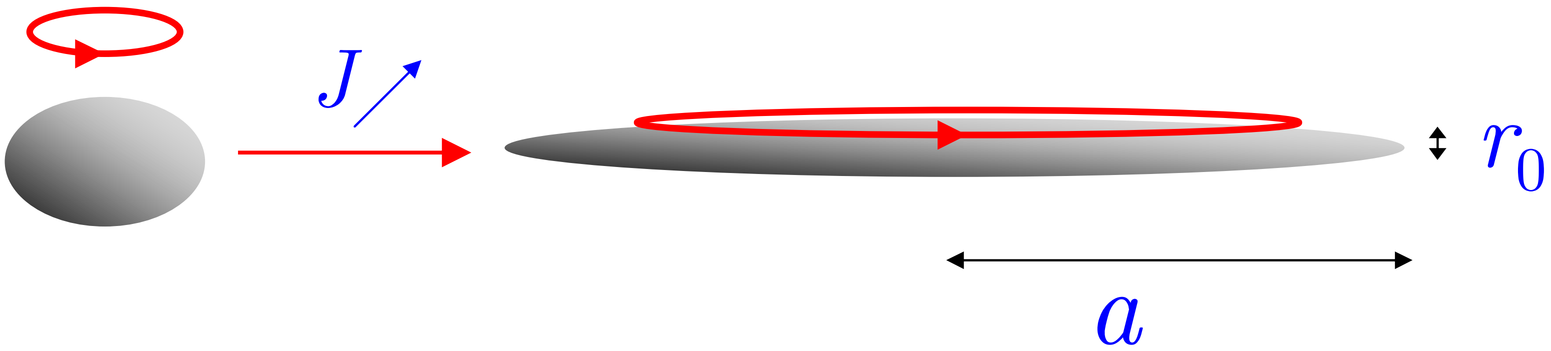
  - No upper bound on J for given M in $D>4$

    $\Rightarrow$ Length scales $\ell_M\,,\ \ell_J$ can differ arbitrarily

- 4D BHs: single scale: $r_0 \sim GM$
  - true even if rotating: Kerr bound $J/M \leqslant GM$
  - no small parameter

Myers-Perry bhs in D⩾6:
Two scales and black brane limit

- Ultra-spinning regime $a \sim J/M \gg (GM)^{1/(D-3)}$

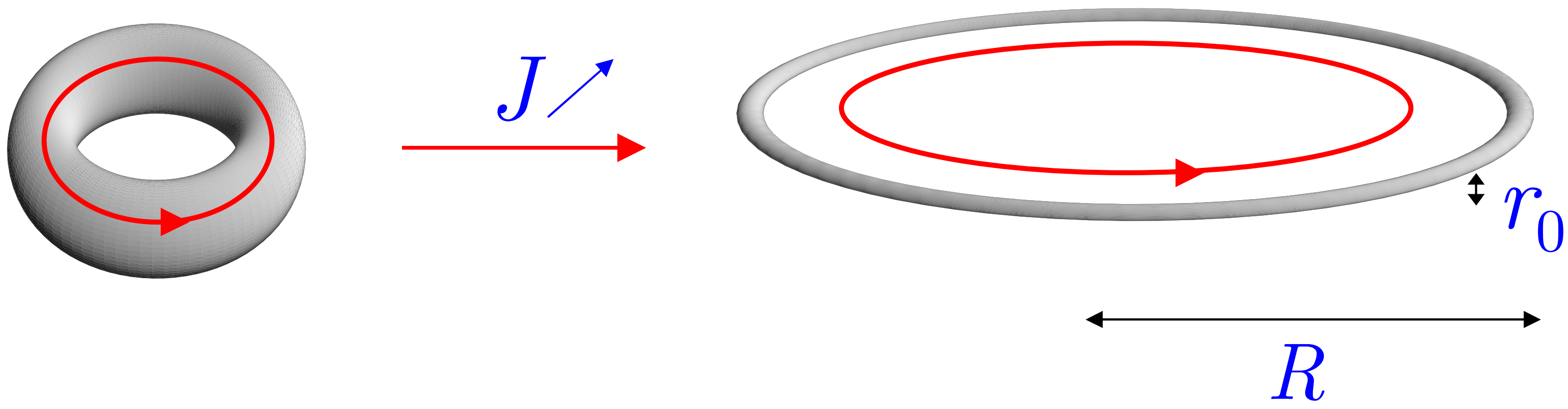


- Limit $a \to \infty$, r_0 finite:

$\Rightarrow$ black 2-brane along rotation plane

Black Ring in D=5
Two scales and black brane limit

- Ultra-spinning regime $R \sim J/M \gg (GM)^{1/(D-3)}$

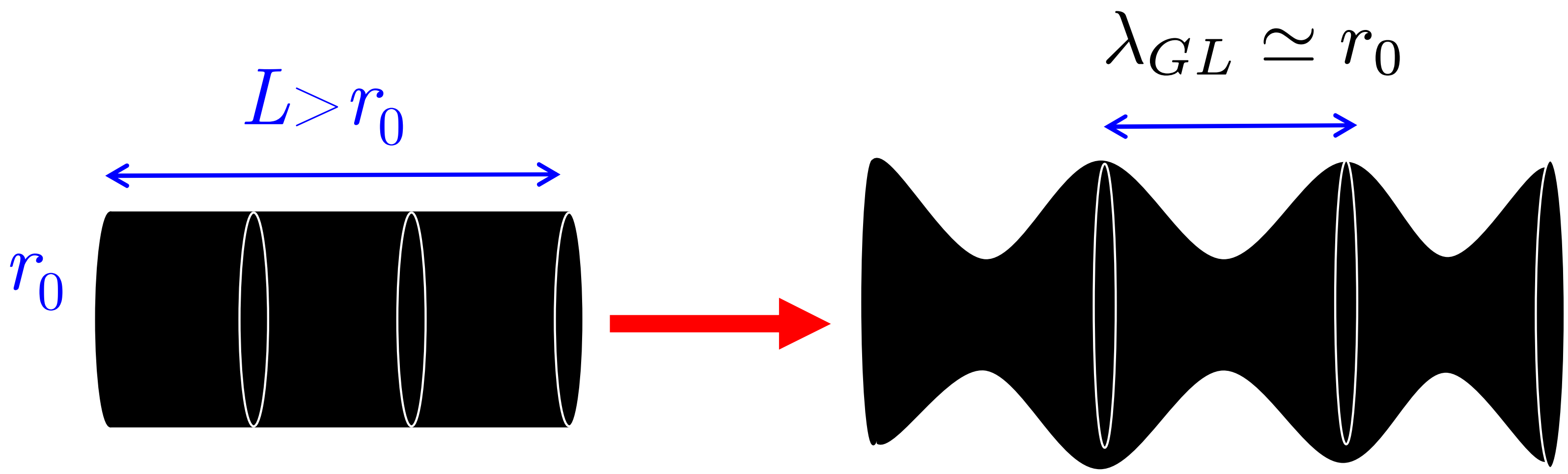


- Limit $R \to \infty$, r_0 finite:

$\Rightarrow$ black string along rotation direction

Also:

- Gregory-Laflamme instability of black brane when the two scales r_0, L begin to differ



$\Rightarrow$ Hi-D bhs have qualitatively new dynamics

unsuspected from experience with 4D bhs

- 4D bhs only possess short-scale ($\sim r_0$) dynamics

- Hi-D bhs: need new tools to deal with long-distance ($\sim R \gg r_0, \quad \ell_J \gg \ell_M$) dynamics

- Natural approach: integrate out short-distance physics, find long-distance effective theory

Effective theory at large length scales

- Separate long- and short-wavelength d.o.f.'s
- Replace short-distance d.o.f.'s with effective theory

$$I_{\mathrm{EH}} = \int \sqrt{-g}R \approx \int_{\lambda \gg r_0} \sqrt{-g_{(\mathrm{long})}}R_{(\mathrm{long})} + I_{\mathrm{eff}}[g_{(\mathrm{long})}, \phi(\sigma)]$$

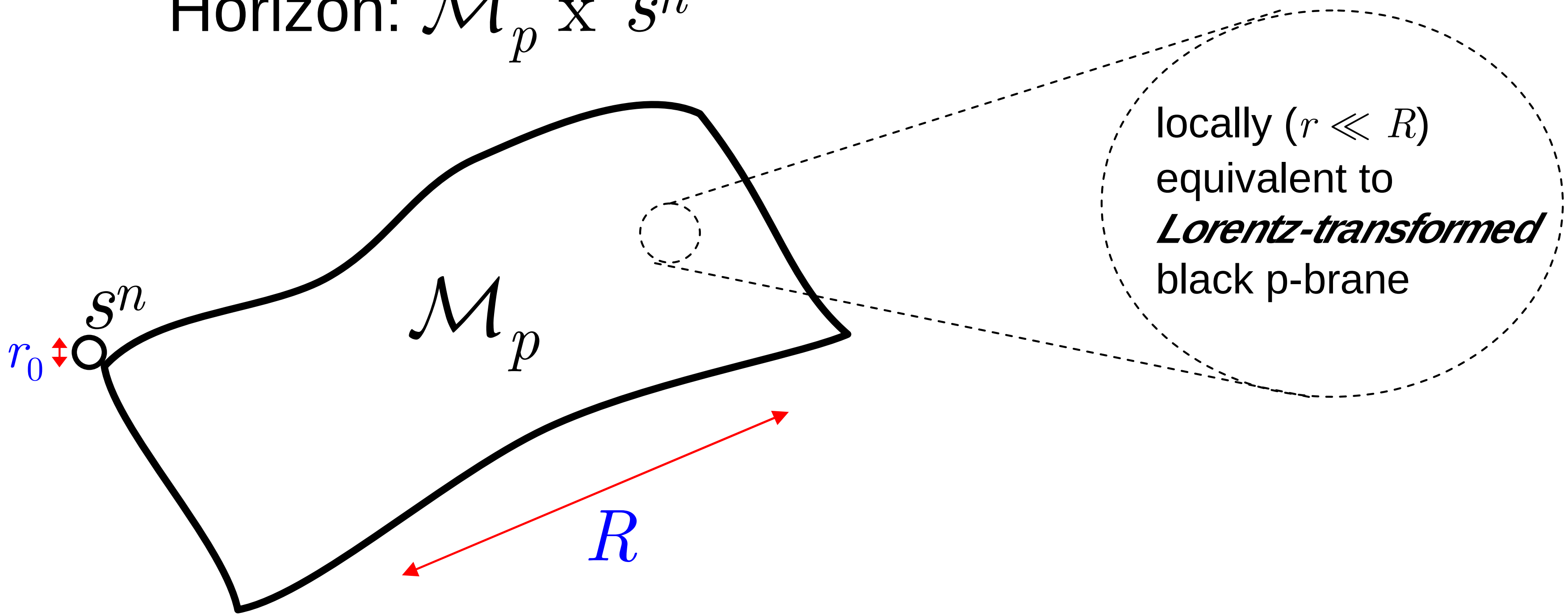
- What kind of effective theory?
  - Hint: limit $\ell_M/\ell_J \to 0$ yields a black brane

$\Rightarrow I_{\mathrm{eff}}$ is a worldvolume theory for the "collective coordinates" $\phi(\sigma)$ of a black brane

Blackfolds: long-distance effective dynamics of hi-d black holes

- ***Black*** p-branes w/ worldvolume = curved submani***fold*** of spacetime

Horizon: $\mathcal{M}_p \times s^n$



- Worldvolume fields (collective coords):
  - $D-p-1$ transverse coordinates $X^{\perp}(\sigma^{\alpha})$
  - Up to p boosts $\Lambda^{0}{}_{\nu}(\sigma^{\alpha})$ (black brane is not boost-invt)
  - 1 thickness $r_0(\sigma^{\alpha})$
- Equations: $T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta I_{\text{eff}}}{\delta g^{\mu\nu}} \quad \rightarrow \quad \nabla_{\mu} T^{\mu\nu} = 0$

  transverse index: $0,\ldots,D-p-1$
- Global *blackness* condition (stationary, regular horizon):
  - Uniform surf gravity κ & angular velocities Ω_i

  → eliminate thickness and boost parameters

- General ***Classical Brane Dynamics*** : ***Carter***

Given any worldvolume source of energy-momentum, in probe approx,

$$\nabla_\mu T^{\mu\rho} = 0 \quad \Rightarrow \quad T^{\mu\nu} K_{\mu\nu}{}^{\rho} = 0$$

extrinsic curvature

or, with external force: $F^\rho = T^{\mu\nu} K_{\mu\nu}{}^{\rho}$

– Newton's force law: $F = ma$

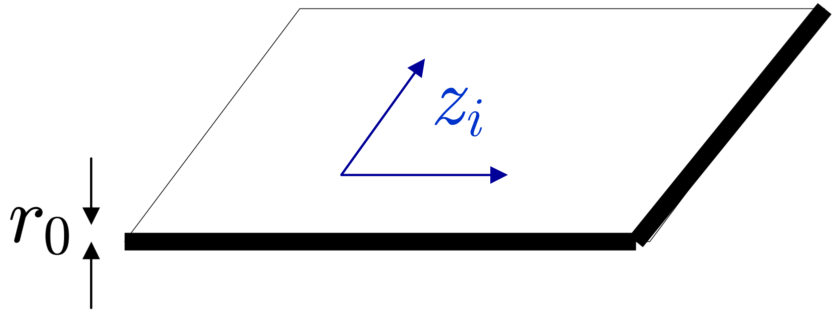
– Nambu-Goto-Dirac eqns: $T_{\mu\nu} = T g_{\mu\nu}$ → $K^\rho = 0$: minimal surface

- What is $T_{\mu\nu}$ for a blackfold?
- Short-distance physics determines effective stress-energy tensor:
  - blackfold locally Lorentz-equivalent to black p-brane of thickness (s^n-size) r_0
  - In region $r_0 \ll r$ field linearizes $\Rightarrow$ approximate brane by equivalent distributional source $T_{\mu\nu}(\sigma^\alpha)$

- Black p-brane

$$ds^2 = -\left(1 - \frac{r_0^n}{r^n}\right) dt^2 + \sum_{i=1}^{p} dz_i^2 + \frac{dr^2}{1 - \frac{r_0^n}{r^n}} + r^2 d\Omega_{n+1}^2$$

$$\begin{aligned} T_{tt} &= r_0^n (n+1) \\ T_{ii} &= -r_0^n \end{aligned}$$



Boost:

$$(t, z_i) = \sigma^\mu \,, \quad \sigma^\mu \to \Lambda^\mu_\nu \sigma^\nu \,, \quad \Lambda^\mu_\nu \in O(1,p)$$

$$T_{\mu\nu} \to T_{\mu\nu} = r_0^n \left[(n+1)\Lambda^t_\mu \Lambda^t_\nu - \textstyle\sum_{i=1}^p \Lambda^i_\mu \Lambda^i_\nu\right]$$

Make $r_0(\sigma^\alpha)$, $\Lambda^\mu{}_\nu(\sigma^\alpha)$ position-dept, and solve for them w/ blackness conds

- Blackness $\Rightarrow T_{\mu\nu}\big(X(\sigma^{\alpha}),\kappa,\Omega_i)\big)$

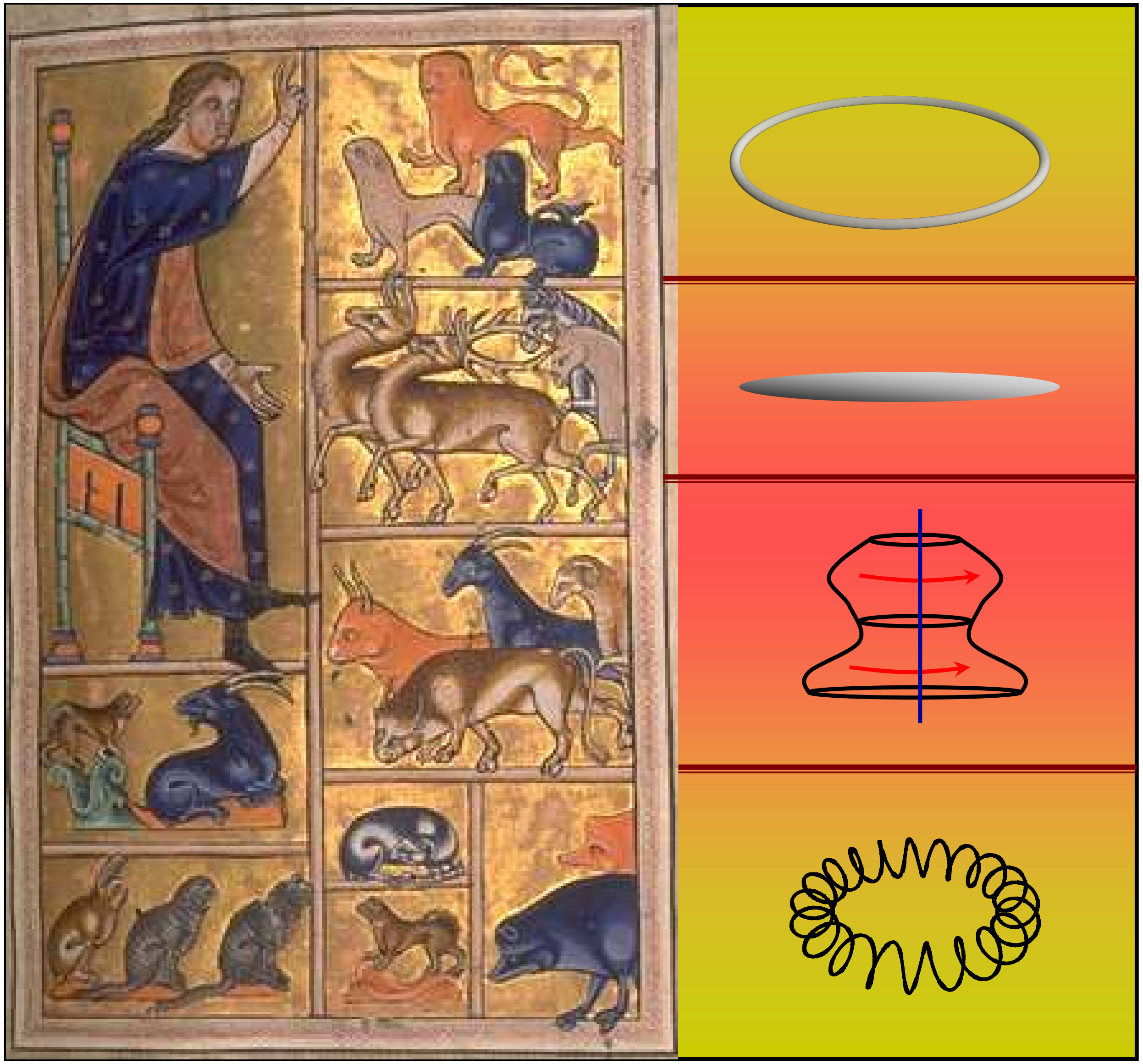
- $K_{\mu\nu}{}^{\rho}\big(X(\sigma^{\alpha})\big)\,T_{\mu\nu}\big(X(\sigma^{\alpha}),\kappa,\Omega_i)\big)=0$

  2nd order diff eqs for wv geometry $X\big(\sigma^{\alpha};\kappa,\Omega_i\big)$

- This is a theory of how black branes can bend

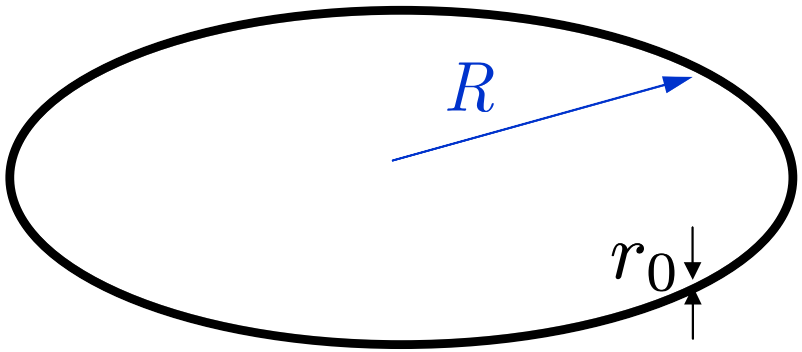
- Similar to Nambu-Goto for cosmic strings, or DBI for D-branes. But:
  - Short-wavelength d.o.f's are gravitational
  - Brane has a horizon. If compact → black hole

Blackfold Bestiary



- Simplest example: black rings in $D \geqslant 5$

$$K_{\mu\nu}{}^{\rho} T^{\mu\nu} = 0$$



$$\Rightarrow \quad \frac{T_{11}}{R} = 0$$

$$T_{11} = r_0^{D-4} \left[(D-4) \sinh^2 \sigma - 1 \right]$$

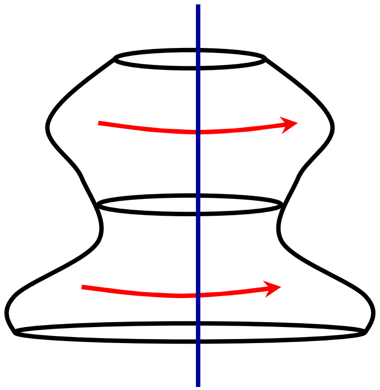
Tune boost to equilibrium $\quad \Rightarrow \sinh^2 \sigma = \dfrac{1}{D-4}$

(in D=5 reproduces value from exact soln)

Horizon S^1 x s^{D-3}

"small" transverse sphere $\sim r_0$

- Axisymmetric blackfolds



(possibly rotations along all axes)

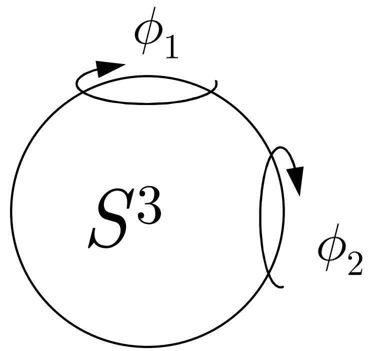
- Simple analytic solutions:
  - even p: ultraspinning MP bh, with $p/2$ ultraspins
  - odd p: round S^p, with all $(p+1)/2$ rotations equal

- I'll illustrate two simple cases of each

- S^3 x s^{n+1} black hole as blackfold

$(n \geqslant 1)$

- Embed three-brane in a space containing



$$ds^2 = dr^2 + r^2 d\Omega^2{}_{(3)}$$

as $r = R$

- Solution exists if $|\Omega_1| = |\Omega_2| = (3/(3+n))^{1/2} R^{-1}$

  size of s^{n+1} $r_0 = \mathrm{const}$

- If $|\Omega_1| > |\Omega_2|$ then numerical solution for

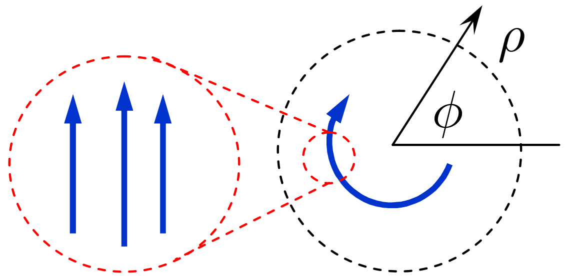
  $r = R(\theta)$: non-round S^3

• Ultra-spinning 6D MP bh as blackfold

- Black two-brane along a plane

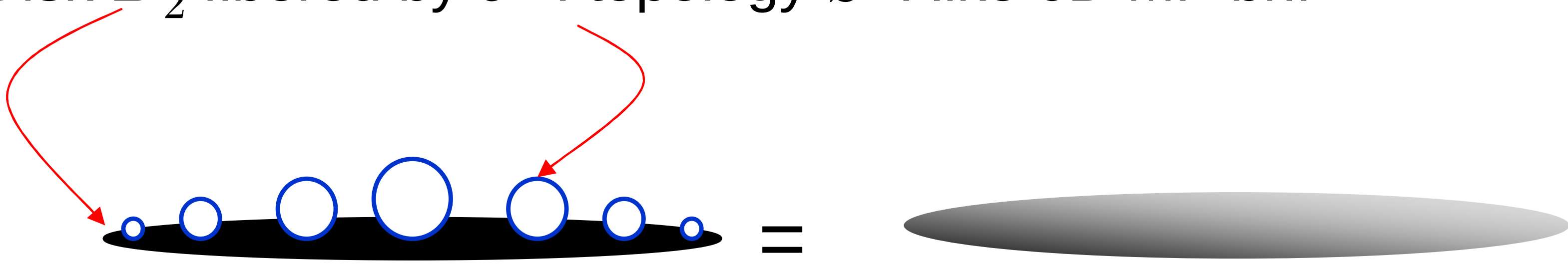
  $ds^2 = d\rho^2 + \rho^2 d\phi^2$

  to obtain planar blackfold $\mathcal{P}_2 \times s^2$



locally equiv to ***boosted black 2-brane***

- Find size $r_0(\rho)$ of s^2 & boost $\alpha(\rho)$ of locally-equiv 2-brane:
  - Soln: $\alpha(\rho) \to \infty, \quad r_0(\rho) \to 0$ at $\rho_{max} = 1/\Omega$
  - Disk D_2 fibered by s^2 : topology S^4: like 6D MP bh!



- ***All physical magnitudes*** match those of the ultraspinning 6D MP bh

- Solving a conjecture on horizon symmetries

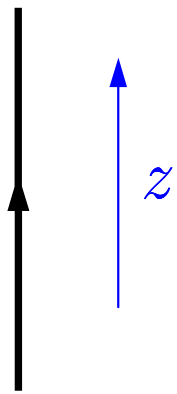
- Rigidity of horizons: How many spatial $U(1)$ isometries must a bh horizon have?
- *Hollands+Ishibashi+Wald*: at least one
- But MP bhs and black rings have much more: all the Cartan subgroup of $O(D-1) \supset U(1)^{\lfloor (D-1)/2 \rfloor}$
  - e.g. 5D bhs have isometry $\mathbb{R}_t \times U(1)_{\phi_1} \times U(1)_{\phi_2}$

- *Reall* conj. (2002): $\exists$ hi-d bhs w/ only $\mathbb{R}_t \times U(1)_\phi$

The solution: Helical blackfolds

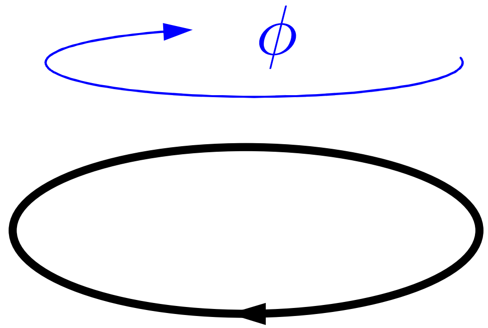
- Place a boosted black string along an isometry ζ of background

$$\zeta = \partial_z$$



black string

$$\zeta = \partial_\phi$$



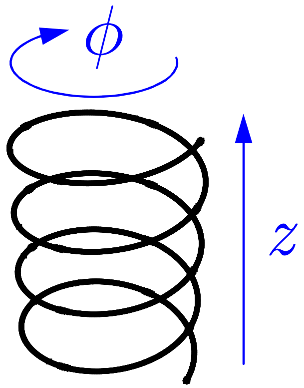
black ring

The solution: Helical blackfolds

- Place a boosted black string along an isometry ζ of background $(D \geqslant 5)$

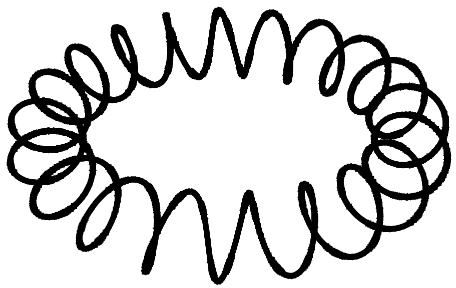
$$\zeta = k\partial_z + \partial_\phi$$

Helical
black string



$$\zeta = n\partial_{\phi_1} + m\partial_{\phi_2}$$

Helical
black ring



(n.b: profile is static!)

- The orthogonal background isometry is broken:
  - horizon has only one spatial U(1) $(D{=}5{,}6)$
  - but bh has two angular momenta (from boost of string)

A programme framework for investigating hi-d black holes

- We don't know the landscape of hi-d bhs in detail yet, but now ***we have a map***
- Black hole dynamics splits into three regimes according to the relative size of scales $\ell_M \sim (GM)^{1/(D-3)}, \quad \ell_J \sim J/M$

1: $\ell_J \lesssim \ell_M$ 2: $\ell_J \sim \ell_M$ 3: $\ell_J \gg \ell_M$

A programme framework for investigating hi-d black holes

- $\ell_J \lesssim \ell_M$: single scale, Kerr-like – not much new expected: uniqueness, stability (classical, linear)
- $\ell_J \sim \ell_M$: threshold of separating scales: GL-like instabilities, inhomogeneous ("pinched") phases, mergers – this is the most difficult to study analytically, but better for numerics
- $\ell_J \gg \ell_M$: separated scales: blackfold dynamics – we have the tools to study it

A programme framework for investigating hi-d black holes

- Change focus:
  - less emphasis on exact solutions
  - search for all D⩾6 black hole solutions in closed analytic form is futile (some may still show up: $p=D-4$)
  - ***classification*** becomes increasingly harder at higher D, but maybe also less interesting

- Black branes are very elastic!

  ➔ Investigate what hi-d black holes and branes can do in specific situations and their novel dynamical possibilities

Ultra-spinning black holes in $D \geqslant 6$

$$ds^2 = -dt^2 + \frac{\mu}{r^{D-5}\Sigma}\left(dt + a\sin^2\theta\, d\phi\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + (r^2 + a^2)\sin^2\theta\, d\phi^2$$

$$+ r^2\cos^2\theta\, d\Omega^2_{(D-4)}$$

Myers+Perry 1986

$$\Sigma = r^2 + a^2\cos^2\theta\,, \qquad \Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}}\,,$$

$$\mu \propto GM$$

$$a \propto \frac{J}{M}$$

$$\frac{\Delta}{r^2} - 1 = -\frac{\mu}{r^{D-3}} + \frac{a^2}{r^2}$$

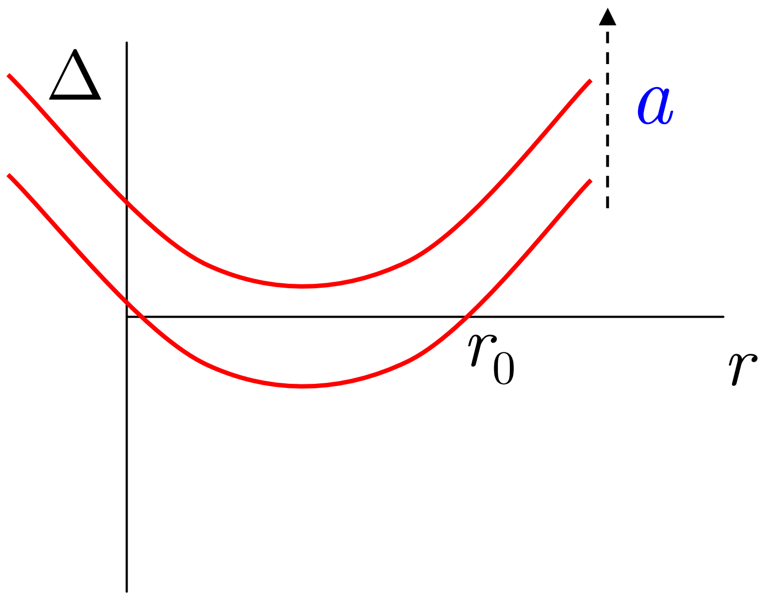
gravitational

centrifugal

$D=4,\ 5$:

Horizon: $\Delta=0$

$$\Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}}$$



Quadratic equation: fix μ, then a can't be too large for real root

4D: $2a \leqslant \mu$

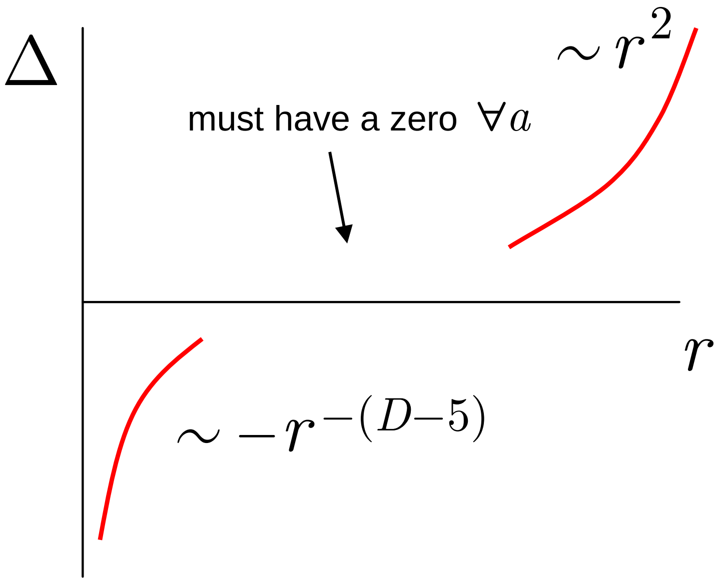
5D: $a^2 \leqslant \mu$

$\Rightarrow$ upper bound on J for given M

$D \geqslant 6$:

Horizon: $\Delta = 0$

$$\Delta = r^2 + a^2 - \frac{\mu}{r^{D-5}}$$



For fixed μ there is an outer event horizon for ***any*** value of a

$\Rightarrow$ No upper bound on J for given M

$\Rightarrow \exists$ ***ultra-spinning black holes***

Blackfold dynamics as 1st Law

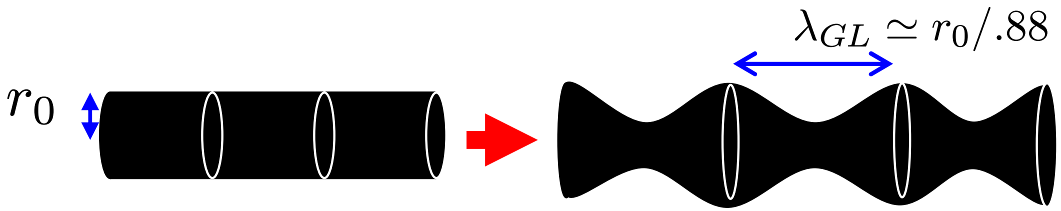
- For stationary blackfolds, compute M, J_i, A_H, by integrating stress-energy tensor T_{tt} , T_{ti} , and horizon area element
- Consider $M\left[x^\mu\right], J_i\left[x^\mu\right], A_H\left[x^\mu\right]$ as functionals of embedding $x^\mu(\sigma^\alpha;\kappa, \Omega_i)$
- Then eqs of motion $K_{\mu\nu}{}^\rho T^{\mu\nu}=0$ are equivalent to

$$\frac{\delta M}{\delta x^\mu} - \frac{\kappa}{8\pi G}\frac{\delta A_H}{\delta x^\mu} - \Omega_i \frac{\delta J_i}{\delta x^\mu} = 0$$

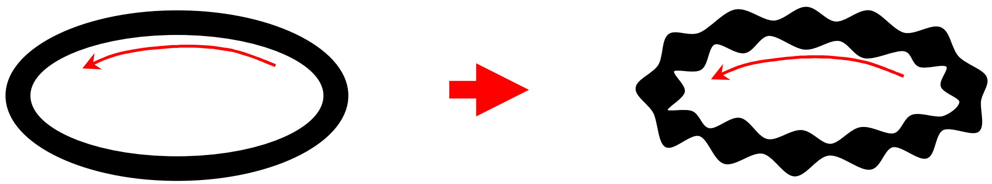
$\Rightarrow$Stationary blackfold eqs = 1st Law

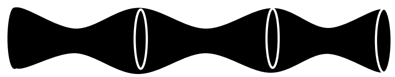
Instabilities and non-uniform phases

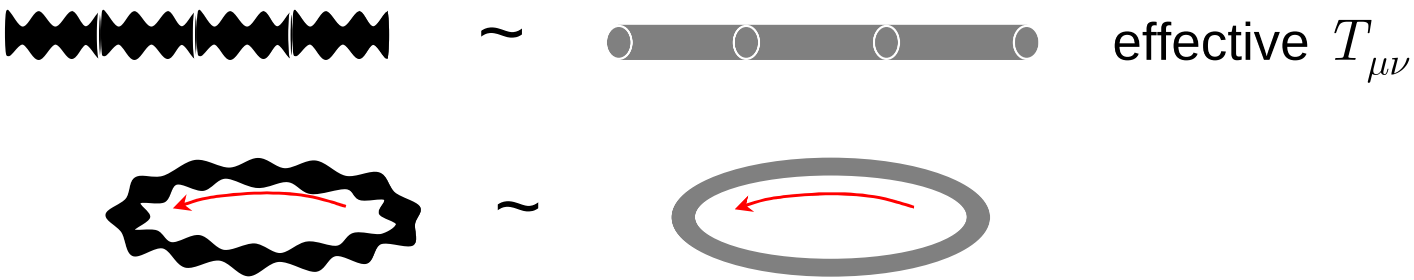
- Stability of blackfolds for long wavelength ($\lambda \gg r_0$) perturbations can be analyzed within blackfold approximation
- But black branes have short-wavelength G-L instabilities



- Expect blackfolds to be unstable – on quick time scales, $\Gamma \sim 1/r_0$



- Non-uniform static black branes exist: 
- Expected to also be unstable below $D_* \ (\sim 13)$ but stable above D_*
- Use stable non-uniform branes as basis for blackfolds (~ wiggly cosmic strings)



- These would emit grav waves, but in a much longer time-scale than GL-instability
$\Rightarrow$ long-lived wiggly blackfolds