Black Hole Hair Removal

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References:

Nabamita Banerjee, Ipsita Mandal, A.S., arXiv:0901.0359

Dileep Jatkar, A.S., Yogesh Srivastava, to appear

Introduction

One of the successes of string theory has been an explanation of the Bekenstein-Hawking entropy of a class of supersymmetric black holes in terms of microscopic quantum states.

$$S_{BH}(Q) = \ln d_{micro}(Q)$$

Strominger, Vafa

$d_{micro}(Q)$: degeneracy of microstates carrying a given set of charges Q.

$$S_{BH}(Q) = A/4G_N$$

This formula is quite remarkable since it relates a geometric quantity in space-time to a counting problem.

However the Bekenstein-Hawking formula for the entropy receives α' and g_s corrections.

α' corrections are described by Wald's formula.

What about quantum corrections?

Our goal is to search for an exact relation of the form

$$\mathsf{d}_{\mathsf{macro}}(\mathsf{Q}) = \mathsf{d}_{\mathsf{micro}}(\mathsf{Q})$$

$\mathsf{d}_{\mathsf{macro}}(\mathsf{Q})$: Some generalization of the Bekenstein-Hawking formula taking into account α' and g_{s} corrections.

We have the best chance of finding such a d_{macro} for extremal (BPS) black holes.

It is in this case that we have a precise definition of d_{micro} – or more accurately an appropriate index – in the microscopic theory.

We shall work in some fixed duality frame so that we can distinguish between classical and quantum effects.

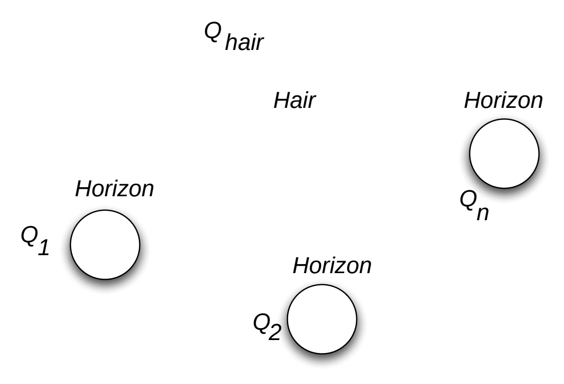
Proposal for d_{macro}:

Take a macroscopic configuration of charge Q.

In general such a configuration could involve an n centered black hole with charges $Q_1, \cdots Q_n$ and hair with charge Q_{hair}.

Hair: smooth normalizable deformations of the black hole solution with support outside the horizon(s).

d_{macro} will receive contribution from both the horizon and the hair.


Q_{hair}
Hair
Horizon
Horizon
Q_1
Horizon
Q_2
Q_n

Proposal for $d_{macro}(Q)$:

$$\sum_{n} \sum_{\substack{\{Q_k\}, Q_{hair} \\ \sum_{k=1}^{n} Q_k + Q_{hair} = Q}} \left\{ \prod_{k=1}^{n} d_{hor}(Q_k) \right\} d_{hair}(Q_{hair}; \{Q_k\})$$

$d_{hor}(Q_{hor})$: contribution from the horizon with charge Q_{hor}

d_{hair}: contribution from the hair of the n-centered black hole, with the horizons carrying charges $Q_1, \cdots Q_n$, and the hair carrying charge Q_{hair}.

d_{hair} can be computed as follows:

1. Identify supersymmetric deformations of the original black hole solution with support outside the horizon.

2. Carry out geometric quantization of these deformations and compute the associated degeneracies.

We shall return to a more detailed discussion of this soon.

d_{hor}: Should be given by some computation in the near horizon $AdS_2 \times K$ geometry of the extremal black hole.

$\longrightarrow$ Quantum Entropy Function.

Although it will not be directly related to our analysis we shall describe this proposal very briefly.

Make a euclidean continuation of the AdS_2 factor and represent it as a Poincare disk.

$$\mathbf{d}_{\mathbf{hor}} = \left\langle \exp[-\mathbf{i}\mathbf{q}_{\mathbf{k}} \oint \mathbf{d}\theta\, \mathbf{A}_{\theta}^{(\mathbf{k})}] \right\rangle^{\mathbf{finite}}$$

$\langle\ \rangle$: Path integral over string fields in the euclidean near horizon background geometry.

$\{\mathbf{q}_{\mathbf{k}}\}$: electric charges carried by the black hole, representing electric flux of the gauge field $\mathbf{A}^{(\mathbf{k})}$ through AdS_2

$\oint$: integration along the boundary of AdS_2

finite: Infrared finite part of the amplitude.

Important point for us:

d_{hor} is determined completely in terms of the near horizon geometry of the black hole.

Thus two black holes with identical near horizon geometry will have identical d_{hor}.

Degeneracy vs. index

Often on the microscopic computation we compute an index rather than absolute degeneracy.

Thus we should also compute the index I_{macro} on the macroscopic side.

Proposed formula for I_{macro}:

$$\sum_{n} \sum_{\substack{\{Q_k\}, Q_{hair} \\ \sum_{k=1}^{n} Q_k + Q_{hair} = Q}} \left\{ \prod_{k=1}^{n} d_{hor}(Q_k) \right\} (-1)^{2J_{hor}} I_{hair}(Q_{hair}; \{Q_k\})$$

I_{hair}: Index of the hair

J_{hor}: total angular momentum associated with the horizon (part of Q_{hor})

A consistency test:

We consider two single centered black holes in type IIB string theory compactified on K3 $\times$ S^1:

1. Rotating charged black hole carrying Q_5 units of D5-brane charge along K3 $\times$ S^1, Q_1 units of D1-brane charge along S^1, n units of momentum along S^1 and equal angular momentum J along the two transverse planes.

$\longrightarrow$ a BMPV black hole. Breckenridge, Myers, Peet, Vafa

2. The same black hole with transverse space Taub-NUT.

$\longrightarrow$ a four dimensional black hole.

Gauntlett, Gutowski, Hull, Pakis, Reall

These two black holes have identical near horizon geometry. Gaiotto, Strominger, Yin; Shih, Strominger, Yin

However the microscopic degeneracies are different.

This difference must be accounted for by the hair degrees of freedom of the two black holes.

Our goal:

1. Explicitly compute the degeneracies associated with the hair degrees of freedom of the two black holes.

2. Remove these hair contributions from the respective microscopic degeneracies.

3. Show that the final results after hair removal are identical for the two black holes.

Partition functions

Note that both the BMPV black hole and the four dimensional black hole are characterized by four quantum numbers Q_1, Q_5, n and J.

The degeneracy depends only on n, J and the combination $N \equiv Q_5(Q_1 - Q_5)$.

Thus in the microscopic analysis we can set $Q_5 = 1$ and analyze the partition function $Z(\rho, \sigma, v)$.

(ρ, σ, v): conjugate to (n, Q_1, J).

Result:

$$Z_{5D}(\rho,\sigma,v) = e^{-2\pi i\rho-2\pi i\sigma}\prod_{\substack{k,l,j\in\mathbf{Z}\\ k\geq 1, l\geq 0}}\left(1-e^{2\pi i(\sigma k+\rho l+vj)}\right)^{-c(4lk-j^2)}$$
$$\times\prod_{l\geq 1}\left\{\left(1-e^{2\pi i(l\rho+v)}\right)^{-2}\left(1-e^{2\pi i(l\rho-v)}\right)^{-2}\right.$$
$$\left.\left(1-e^{2\pi il\rho}\right)^{4}\right\}(-1)\left(e^{\pi iv}-e^{-\pi iv}\right)^{2}.$$

$$Z_{4D}(\rho,\sigma,v) = -e^{-2\pi i\rho-2\pi i\sigma-2\pi iv}$$
$$\prod_{\substack{k,l,j\in\mathbf{Z}\\ k,l\geq 0, j<0 \text{ for } k=l=0}}\left(1-e^{2\pi i(\sigma k+\rho l+vj)}\right)^{-c(4lk-j^2)}.$$

Dijkgraaf, Verlinde, Verlinde

The coefficients c(n) are defined via

$$8\left[\frac{\vartheta_2(\tau,z)^2}{\vartheta_2(\tau,0)^2}+\frac{\vartheta_3(\tau,z)^2}{\vartheta_3(\tau,0)^2}+\frac{\vartheta_4(\tau,z)^2}{\vartheta_4(\tau,0)^2}\right]=\sum_{j,n\in\mathbf{Z}} c(4n-j^2)\, e^{2\pi i n\tau+2\pi i j z}$$

The starting point of both the four and five dimensional black holes is the elliptic genus of symmetric product of K3's, describing the degeneracies associated with the relative motion between the D1 and D5-branes.

Dijkgraaf, Moore, Verlinde, Verlinde

Z_{5D} and Z_{4D} are obtained by multiplying it by the partition function associated with the additional degrees of freedom of the system.

Task

1. Calculate the partition function Z_{5D}^{hair} associated with the hair degrees of freedom of the 5D black hole.

2. Calculate the partition function Z_{4D}^{hair} associated with the hair degrees of freedom of the 4D black hole.

Compare Z_{5D}/Z_{5D}^{hair} with Z_{4D}/Z_{4D}^{hair}.

Hair removal

Hair of five dimensional black hole:

1. Normalizable plane wave of gravitons describing transverse oscillation of the system.$**$

– characterized by four independent functions of $(t + y)$

t: time y: coordinate along S^1

2. Normalizable plane wave like excitations of the gravitino.

– characterized by four independent functions of $(t + y)$

3. Some additional fermion zero modes associated with broken supersymmetry.

All these deformations have been constructed explicitly as classical solutions of the supergravity equations of motion.

∗∗: The graviton plane wave modes have curvature singularity at the future event horizon.

Horowitz, Yang; Kaloper, Myers, Roussel

Thus we should not count them as true hair degrees of freedom.

Result for the hair partition function:

$$Z_{5D}^{hair} = (e^{\pi i v} - e^{-\pi i v})^4 \prod_{l \geq 1} (1 - e^{2\pi i l \rho})^4 \,.$$

Singularity free hair of four dimensional black hole:

1. Normalizable plane wave of gravitons describing transverse oscillation of the system.

– characterized by 3 independent functions of $(t+y)$

2. Plane wave like excitations of the self-dual 2-form fields associated with the normalizable harmonic 2-form of the Taub-NUT space.

– characterized by 21 independent functions of $(t+y)$

3. Normalizable plane wave like excitations of the gravitino.

– characterized by 4 independent functions of $(t+y)$

4. Some additional fermion zero modes.

$$Z^{hair}_{4D}(\rho, \sigma, v) = \prod_{l=1}^{\infty} \left[\left(1 - e^{2\pi i l \rho}\right)^{-20}\right.$$

$$
\begin{aligned}
Z_{5D}/Z_{5D}^{\text{hair}} &= -e^{-2\pi i\rho-2\pi i\sigma}\left(e^{\pi i v}-e^{-\pi i v}\right)^{-2} \\
&\quad \prod_{\substack{k,l,j\in\mathbb{Z}\\ k\geq 1, l\geq 0}}\left(1-e^{2\pi i(\sigma k+\rho l+vj)}\right)^{-c(4lk-j^2)} \\
&\quad \left\{\prod_{l\geq 1}\left(1-e^{2\pi i(l\rho+v)}\right)^{-2}\left(1-e^{2\pi i(l\rho-v)}\right)^{-2}\right\}
\end{aligned}
$$

$$Z_{4D}/Z_{4D}^{\text{hair}} = \text{same as above}$$

Thus the two results match, as is expected from identification of the near horizon geometries of the two black holes.

Conclusion

Our results indicate that two black holes with the same near horizon geometry have identical microscopic degeneracies after we remove the contribution to the degeneracies from the hair degrees of freedom of the black hole.

This is consistent with the hypothesis that we can associate a degeneracy to the horizon of the black hole that can be expressed as some computation in the near horizon geometry of the black hole without any reference to the full black hole solution.