Type IIB GUT vacua and their F-theory uplift

based on

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Motivation

GUT model building classic topos in string phenomenology since 1985

new impulses from recent GUT model building advances in F-theory

Donagi/Wijnholt and Beasley/Heckman/Vafa 2008

F-theory:

- IIB compactification with D-branes on fully backreacted manifold
- sensitive to non-perturbative effects beyond Type IIB

work on F-theory GUTs so far: properties of local quivers

except: Tatar et al.'08; Andreas,Curio; Donagi,Wijnholt; Marsano et al. '09

Global consistency conditions are at the heart of string theory: **distinguish string landscape from swampland**

at present consistency conditions better understood in Type IIB language (esp. gauge flux)

most promising avenue for unification from F-theory is via SU(5) GUT
$\Rightarrow$ This is (and has been for a while) amenable to Type II methods

Motivation

This talk:
Systematic analysis of SU(5) GUTs in IIB vacua

Aim: implementation of GUT quivers into **actual string vacua as opposed to local quivers**

- Explicit **Type IIB vacua serve as starting point for F-theory** models upon uplifting
  Do useful geometric properties of Type IIB Calabi-Yau 3-fold survive?
- Ultimate goal:
  model building in combination with moduli stablisation
  $\leftrightarrow$ required for satisfactory discussion of SUSY breaking, predictions...
  Type IIB orientifolds on genuine (conformal) Calabi-Yau promising

Outline

1) Motivation

2) Background on Type IIB orientifolds with D3/D7-branes

- gauge flux
- global consistency conditions

3) SU(5) GUT model building in Type IIB orientifolds

- GUT breaking
- non-perturbative Yukawa couplings

4) Explicit construction of semi-realistic SU(5) vacua

5) F-theory uplift

- 4-folds from 3-folds
- gauge enhancements: IIB vs. F

5) Conclusions

Type IIB Orientifolds

- Compactification of Type IIB theory on Calabi-Yau X
- orientifold: divide by $\Omega(-1)^{F_L}\sigma$, σ : holomorphic involution of X

$$\sigma^* J = J, \quad \sigma^*\Omega = -\Omega$$

split into even/odd cycles : Grimm, Louis '05

$h^{1,1}_+$ Kähler moduli $T_I = \int_{\gamma_I^+} e^{-\phi} J \wedge J + iC_4$,

$h^{1,1}_-$ B-field moduli $G_i = \int_{\gamma_i^-} -B + iC_2$

discrete B-field parameter $\frac{1}{2\pi}\int_{\gamma_i^+} B = 0, \frac{1}{2}$

$\Rightarrow$ fix-point set O3/O7-planes $\leftrightarrow$ spacetime-filling D3/D7-branes

- D3-brane: point on internal X
- D7-brane: wraps holomorphic 4-cycle (divisor) D_a

upstairs geometry: D_a + image D'_a

1) D_a not invariant: $D_a \rightarrow D'_a \Rightarrow$ gauge group $U(N_a)$

2) D_a invariant: $2N_a \Rightarrow SO(2N_a)/Sp(2N_a)$

Gauge Flux

Gauge flux on D-brane: $\mathcal{F}_a = F_a + \iota^* B$

focus on $\langle \mathcal{F}_a \rangle \neq 0$ for abelian subgroups $\Leftrightarrow$ line bundles L_a

- typical embedding: diagonal $U(1) \subset U(N) \rightarrow SU(N) \times U(1)$,
  $U(1)$ massive by Green-Schwarz mechanism
- can also switch on $U(1) \subset SU(N)$

gauge flux on divisor $D \Leftrightarrow \langle F \rangle \in H^2(D) \Leftrightarrow$ 2-cycle $\in H_2(D)$

2 types of non-trivial 2-cycles on D: Lerche, Mayr, Warner '01/02;

non-trivial also on X vs. boundaries of 3-chains on X Jockers,Louis'05

splitting $$L_a = \underbrace{\iota^* \mathbb{L}_a}_{\text{pullback from X}} \otimes \underbrace{R_a}_{\text{trivial on X}}$$

flux R_a does

- not affect chiral spectrum and
- not participate in GS mechanism Buican et al. 2006

Global consistency conditions (I)

1) Freed-Witten quantisation condition on line bundles

path-integral of open string worldsheet with boundary on single U(1) brane D must be well-defined   Freed, Witten '99

Result: shift in Dirac quantisation condition

$c_1(L) - \iota^* B + \frac{1}{2} c_1(K_D) \in H^2(D, \mathbb{Z})$

$\Rightarrow$ L half-integer quantised

- for discrete B-field $\int_{\gamma_+} B = \frac{1}{2}$
- for divisor D not spin, i.e. $c_1(K_D) \in H^2(D, (2\mathbb{Z}+1)/2)$

choice of B-field determines quantisation on several divisors at once!

Generalisation to more general embedding: Blumenhagen,Braun,Grimm,T.W. '08

$$T_0\,(c_1(L_a^{(0)}) - \iota^* B) + \sum_i T_i\, c_1(L_a^{(i)}) + \frac{1}{2} T_0\, c_1(K_{D_a}) \in H^2(D_a, \mathbb{Z})_{N_a \times N_a}$$

$\Rightarrow$ suitably fractional line bundles are allowed!

Global consistency conditions (II)

2) Tadpole cancellation condition from CS action of D-brane and O-plane

- cancellation of D7-charge and induced D5-charge
- D3 :

$$N_{\rm D3} + \frac{N_{\rm flux}}{2} - \sum_a \int_{D_a} \frac{{\rm tr}\,\mathcal{F}_a^2}{8\pi^2} = \underbrace{\frac{N_{\rm O3}}{4} + \frac{\chi(D_{\rm O7})}{12} + \sum_a N_a \frac{\chi_o(D_a)}{24}}_{\chi(CY_4)/12\,{\rm in\,F-theory}}$$

constraint by integrality of $N_{D3} \in \mathbb{Z}_0^+ \leftrightarrow$ Freed-Witten quantisation!

models with non-spin divisors tricky!

3) D-term supersymmetry

Fayet-Iliopoulos D-term: $\xi \;\sim\; \int_D \iota^* J \wedge c_1(L)$ MMMS '99;

$\xi_a = 0 \rightarrow -\frac{N_a}{2} \int_{D_a} c_1^2(L_a) \geq 0$ Blumenhagen,Braun,Grimm,T.W. '08

$\Rightarrow$ SUSY bundles always contribute positively to D3-tadpole

$\leftrightarrow$ danger of overshooting

Massless Matter

adjoint chiral fields: $h^{(0,2)}(D)$ deformation and $h^{(0,1)}(D)$ Wilson moduli

charged chiral matter at intersection of D-branes along divisors D_a and D_b

open strings in

- $a \longrightarrow b$ sector: bifundamental matter $(\overline{\square}_{a(-1)}, \square_{b(1)})$
- $a' \longrightarrow a$ sector: (anti-)symmetric matter $\begin{smallmatrix}\square\\\square\end{smallmatrix}_{(2)}$ / $\square\square_{(2)}$

  - $D_a = D_b \rightarrow$ matter localised on whole divisor D_a
    $\exists$ generically vector-like pairs
  - $D_a \neq D_b \rightarrow$ matter localised on curve $C_{ab} = D_a \cap D_b$
    generically no vector-like pairs

either case: index $I_{ab} = -\int_Y [D_a] \wedge [D_b] \wedge (\, c_1(L_a) - c_1(L_b)\,)$

relative flux only affects vector-like spectrum

SU(5) GUTs

starting point: $U(5)_a \times U(1)_b$ theory $\qquad$ $U(1)_{a,b}$ massive

GUT brane: (D_a, L_a), $\qquad$ U(1) brane (D_b, L_b)

sector	reps.	particle	sector	reps.	particle
(a', a)	$\mathbf{10}_{(2,0)}$	(Q_L, u^c_R, e^c_R)	(b', b)	$\mathbf{1}_{(0,2)}$	N^c_R
(a, b')	$\overline{\mathbf{5}}_{(-1,-1)}$	(d^c_R, L)	(a, b)	$\mathbf{5}^H_{(1,-1)} + \overline{\mathbf{5}}^H_{(-1,1)}$	$(T^u, H^u) + (T^d, H^d)$

matter localised on intersection loci

$\mathbf{10}_{(2,0)} \quad \Longleftrightarrow \quad H^*(D_a \cap D_{a'}, L_a^2)$

$\overline{5}^m_{(-1,-1)} \quad \Longleftrightarrow \quad H^*(D_a \cap D_{b'}, L_a^{-1} \otimes L_b^{-1})$

Two major challenges:

- complete description of GUT breaking:
  same solution exists in IIB/F-theory
  cannot be treated locally - depends on global data!

- Yukawa couplings:
  distinct approaches in IIB/ F-theory

SU(5) GUT breaking

Idea: **Embed** $U(1)_Y \subset U(5)$ [Beasley,Heckman,Vafa; Donagi,Wijnholt '08]

Approach very sensitive to global consistency:

1) $U(1)_Y$ massless iff $\mathcal{L}_Y$ in relative cohomology

rigid GUT divisor must possess 2-cycles trivial on ambient space

global feature - depends on compactification details, not on local data!

2) Freed-Witten quantisation: Blumenhagen,Braun,Grimm,T.W. '08

$\mathcal{L}_a \leftrightarrow T_a, \quad \mathcal{L}_Y \leftrightarrow \frac{2}{5}T_a + \frac{1}{5}T_Y \qquad T_a = 1_{5\times 5}, T_Y = \mathrm{diag}(-2,-2,-2,3,3)$

GUT breaking: $U(5)_a \to SU(3) \times SU(2) \times U(1)_Y \times U(1)_a$

From general quantisation condition:

$c_1(\mathcal{L}_a) - \iota^* B + \frac{1}{2}K_{D_a} \in \mathbb{Z}, \quad c_1(\mathcal{L}_a) + c_1(\mathcal{L}_Y) - \iota^* B + \frac{1}{2}K_{D_a} \in \mathbb{Z}$

- for non-spin divisor D_a and $B = 0$:
  $\mathcal{L}_a \neq \mathcal{O}$ such that $\mathcal{L}_a$ half-integer quantised
- $\mathcal{L}_Y$ integer quantised

SU(5) GUTs

Yukawa couplings from triple intersection of 3 matter curves

Problem: $\mathbf{10}^{(2,0)}\,\mathbf{10}^{(2,0)}\,\mathbf{5_H}^{(1,-1)}$ forbidden perturbatively in Type IIB

- **Solution: Stringy D-brane instantons** Blumenhagen,Cvetič,Weigand; Ibanez,Uranga; Florea,Kachru,McGreevy,Saulina '06

  Euclidean D3-brane along divisor Ξ with $\Xi \cap D_{a,b} \neq 0$
  $\rightsquigarrow$ charged fermionic zero modes λ_a^i, λ_b^j induce coupling

  $W_{n.p.} \ni Y_\alpha\, Y_\beta\;\; \mathbf{10}^\alpha\, \mathbf{10}^\beta\, \mathbf{5}_H\;\, e^{-\frac{\mathrm{Vol_E}}{g_s}}$ if $I_{a,\Xi} = 1 = I_{b,\Xi}$

  Blumenhagen,Cvetič,Lüst,Richter,Weigand 2007

- **Drawback:** realistic GUT models in Type II require
  $S_{\mathrm{inst.}} \simeq \frac{\mathrm{Vol_E}}{g_s} \longrightarrow 0$

  Philosophy: Search for setup where by classical D-terms $Vol_{\mathrm{E}} = 0$
  quantum corrections will resolve this at $Vol_{\mathrm{E}} = \mathcal{O}(l_s)$
  Note: GUT brane can still be large!

Summary of approach

General requirements on compact CY:

- divisor D_a with $h^{(0,1)}(D_a) = 0 = h^{(0,2)}(D_a)$ for GUT brane
- existence of relative two-cycles on D_a for $\mathcal{L}_Y$
- additional divisor D_b with intersection $D_a \cap D_b$
- define orientifold action, preferrably such that $D_{a'} \neq D_a$

SU(5) property	mechanism
no vector-like matter	localisation on curves
1 vector-like of Higgs	choice of line bundles
3-2 splitting	Wilson lines on $g = 1$ curve
3-2 split + no dim=5 p^+-decay	local. of H_u, H_d on disjoint comp.
$\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}}_H$ Yukawa	perturb. or D3-instanton
$\mathbf{10}\,\mathbf{10}\,\mathbf{5}_H$ Yukawa	presence of appropriate D3-instanton

Explicit constructions

del Pezzo transitions of quintic $Q = \mathbb{P}^4[5], \quad (h^{1,1} = 1, \quad h^{2,1} = 101)$

1st step in chain of transistions: $Q \to Q^{dP_6}$

- create dP_6 singularity by fixing some complex structure moduli
- blow up singularity by pasting in a dP_6

$\Rightarrow h^{1,1}(Q^{dP_6}) = 2, \quad h^{2,1}(Q^{dP_6}) = 90$

$\Rightarrow$ **del Pezzo rigid** ✓

$\Rightarrow$ **ingredients for massless $U(1)_Y$** ✓

del Pezzo: $\mathbb{P}^2$ with n points blown up to a $\mathbb{P}^1$ curve E_i,
$H^{1,1}(dP_n) = \langle l, E_1, \dots E_n \rangle, \quad l \cdot l = 1 = -E_i \cdot E_i$
canonical class $K = \mathcal{O}(f), f = -3l + \sum_i E_i$

$h^{1,1}(Q^{dP_6}) = 2 \Rightarrow$ only f is non-trivial on Q^{dP_6}
trivial ones: those orthogonal to f: $\langle l - E_1 - E_2 - E_3, E_i - E_j + 1 \rangle$

$c_1(\mathcal{L}_Y) = E_i - E_j$ leads to no vectorlike exotics from breaking of $\mathbf{24}$

Explicit constructions

toric description to analyse intersection form and topology of divisors

scaling relations: $\{x_i\} \simeq \{\lambda^{Q_1(x_i)} x_i\} \simeq \{\mu^{Q_2(x_i)} x_i\}$

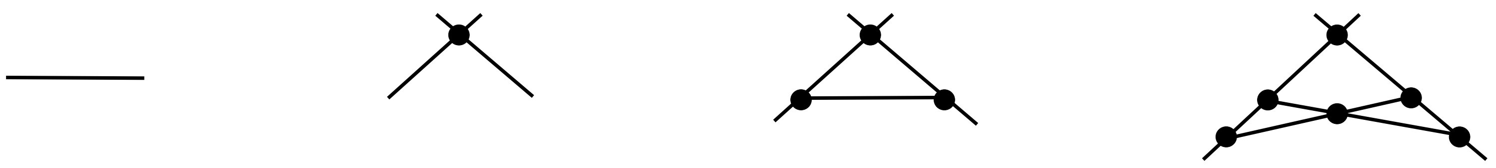
	u_1	u_2	u_3	u_4	v	w
Q_1	1	1	1	1	1	0
Q_2	0	0	0	0	1	1
class	H	H	H	H	H +X	X

Q^{dP_6} :

$P_{(5,2)}(u_i, v, w) = 0$

sequence of transitions: fix more compl. structure and blow-up

Q^{dP_6} $\quad Q^{(\mathrm{dP}_7)^2}$ $\quad Q^{(\mathrm{dP}_8)^3}$ $\quad Q^{(\mathrm{dP}_9)^4}$



- new del Pezzos intersect in $\mathbb{P}^1$ $\Rightarrow$ matter curves
- E_6 sublattice of each higher dP_n is trivial on Calabi-Yau $\Rightarrow$ $U(1)_Y$

Explicit constructions

2 types of involutions: inversion: $x_i \to -x_i$ or exchange : $x_i \leftrightarrow x_j$

Focus on exchange involution on $Q^{dP_9^4}$:

	z	u_1	u_2	v_1	v_2	w_1	w_2	x_1	x_2
Q_1	1	1	1	1	1	0	0	0	0
Q_2	0	0	0	0	**1**	**1**	0	0	0
Q_3	0	0	0	**1**	0	0	**1**	0	0
Q_4	0	0	**1**	0	0	0	0	**1**	0
Q_5	0	**1**	0	0	0	0	0	0	**1**
class	D_5	$D_5{+}D_9$	$D_5{+}D_6$	$D_5{+}D_8$	$D_5{+}D_7$	D_7	D_8	D_6	D_9

Involution: $\sigma : v_1 \leftrightarrow v_2, \qquad w_1 \leftrightarrow w_2$

invariant: $v_1 v_2, \quad w_1 w_2, \quad v_1 w_1 + v_2 w_2,$ anti-inv.: $v_1 w_1 - v_2 w_2$

$h_+^{1,1} = 4, \qquad h_-^{1,1} = 1$

O7-plane: $v_1 v_2 - w_1 w_2 = 0, \qquad [O7] = [D_5 + D_7 + D_8] \,, \qquad \chi(O7) = 37$

further: $N_{03} = 3 \Rightarrow N_{03} + \chi(O7) = 40$

Explicit constructions

SU(5) GUT stack on dP_9:

$$
\begin{aligned}
U(5):&\quad D_a = D_7, & D'_a &= D_8,\\
U(1):&\quad D_b = D_5, & D'_b &= D_5,\\
U(3):&\quad D_c, = D_5 + D_7, & D'_c &= D_5 + D_8
\end{aligned}
$$

D7-TAD ✓

matter curves:

$\mathbf{10}$: $D_7 \cap D_8 = \mathbb{P}^1$ $\quad$ $\mathbf{5_m}$: $D_7 \cap D_5 = T^2$ $\quad$ $\mathbf{5_H} + \overline{\mathbf{5}}_\mathbf{H}$: $D_8 \cap D_5 = T^2$

find line bundles + B-field that are quantised properly
(Freed-Witten: divisors are non-Spin!)

possible to obtain exactly 3 generations, 1 vectorlike Higgs pair, no exotics

Drawbacks:

- Overshooting of D3-TAD by 3 units
- uncertainty of one K-theory constraint

Semi-realistic global example

Example on manifold $Q^{dP_9^4}$: GUT brane on dP_9

[Blumenhagen,Braun,Grimm,Weigand 0811.2936]

property	mechanism	status
globally consistent	tadpoles + K-theory	$\checkmark^{*,**}$
D-term susy	vanishing FI-terms inside Kähler cone	$\checkmark$
gauge group $SU(5)$	$U(5) \times U(1)$ stacks	$\checkmark$
3 chiral generations	choice of line bundles	$\checkmark$
no vector-like matter	localisation on $g = 0, 1$ curves	$\checkmark$
5 vector-like Higgs	choice of line bundles	$\checkmark$
no adjoints	rigid 4-cycles, del Pezzo	$\checkmark$
GUT breaking	$U(1)_Y$ flux on trivial 2-cycles	$\checkmark$
3-2 splitting	Wilson lines on $g = 1$ curve	$\checkmark$
3-2 split + no dim=5 p-decay	local. of H_u, H_d on disjoint comp.	—
$\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}}_{\mathbf{H}}$ Yukawa	perturbative	$\checkmark$
$\mathbf{10}\,\mathbf{10}\,\mathbf{5}_{\mathbf{H}}$ Yukawa	presence of appropriate D3-instanton	$-^{***}$

F-Theory uplift

F-Theory: Elliptic fibration Y over base B

$y^2 = x^3 + x\,z^4\,f(\mathbf{u}) + z^6\,g(\mathbf{u})$, $\quad f \in H^0(B, K_B^{-4})$, $g \in H^0(B, K_B^{-6})$

Branes $\leftrightarrow$ Kodaira degenerations of fiber at $\Delta = 4f^3 + 27g^2 = 0$

Calabi-Yau condition for $Y \leftrightarrow$ D7-Tadpole

$c_1(K_Y) = 0 \leftrightarrow \sum_i c_i\,D_i = 12\,c_1(B)$, $\quad c_i$: degree of zeroes of Δ

Distinguish 2 cases:

- Y is smooth $\leftrightarrow$ B is Fano $\;(-K_B|_C > 0 \quad \forall$ curves $C)$
  $\Rightarrow$ only I_1 degenerations of fiber $\leftrightarrow$ no non-abelian gauge groups
  Euler character: $\chi^*(Y) = 12\int_B c_1(B)\,c_2(B) + 360\int_B c_1^3(B)$

  [Sethi,Vafa,Witten '96]

- Non-abelian enhancement for proper singularities in Y
  Consequence: $\chi(Y) = \chi^*(Y) - \delta$
  E.g. for singularities only in codimension 1 along D:
  $\chi(Y) = \chi^*(Y) - r_G\,c_G\,(c_G + 1)\int_D c_1(D)^2$

[Klemm et al '97; Andreas,Curio '99/'09]

F-Theory uplift

Connection to IIB orientifold on CY 3-fold X by **Sen limit**: [Sen '96/97]

general ansatz: $f = -3h^2 + \epsilon\eta, \qquad g = -2h^3 + \epsilon h\eta - \frac{\epsilon^2}{12}\chi$

IIB limit: $\epsilon \to 0 \Rightarrow \Delta = -9\epsilon^2 h^2(\eta^2 - h\chi) + \mathcal{O}(\epsilon^3)$

$$O7 : h = 0, \qquad D7 : \eta^2 - h\chi = 0$$

X: double cover of base B branched over $h = 0$
Simplest case: X given by equation $h = \xi^2$, orientifold $\xi \to -\xi$

Uplift: Reversal of Sen limit [Collinucci: 0812.0175, 0906.0003]; [Blumenhagen,Grimm,Jurke, TW 0906.0013]

- Define $B = X/\sigma$ and consider Weierstrass model thereof
- Check: compare $\chi(Y)/24 \leftrightarrow \frac{\chi(D_{O7})}{12} + \sum_a N_a \frac{\chi(D_a)}{24}$
  for configuration of D7-branes on top of O7-plane

  $\chi(Y)$ has to take into account possible singularities reflecting non-abelian gauge groups in IIB (if present)

F-Theory uplift

First example: Single Del Pezzo transition of Quintic

Quintic $\leftrightarrow u_1, \dots u_4, v$ $\quad dP_6$: $D_w = [w = 0]$

	u_1	u_2	u_3	u_4	v	w	
Q_1	1	1	1	1	1	0	5
Q_2	0	0	0	0	1	1	2
class	H	H	H	H	H +X	X	

Calabi-Yau condition: $X : P_{(5,2)}(u_i, v, w) = 0$

Involution: $\sigma : v \to -v$

Requires that def. Polynomial contains even powers of v, w

$P_{5,2} = p(u_i)_3\, v^2 + q(u_i)_5\, w^2$

Fixed point set:

$(u_i, v, w) = (u_i, 0, w) \cup (u_i, v, w) = (u_i, v, 0) \equiv (u_i, -v, 0)$

$O7 = D_v + D_w$

F-Theory uplift

Construction of $B = X/\sigma$:

2-1 map $X \to B$: $(u_i, v, w) \mapsto (u_i, v^2, w^2) \equiv (u_i, \tilde{v}, \tilde{w})$

$P_{5,2} = p(u_i)_3\, v^2 + q(u_i)_5\, w^2 \to Q_{5,1}(u_i, \tilde{v}, \tilde{w}) = p(u_i)_3\, \tilde{v} + q(u_i)_5\, \tilde{w}$

	u_1	u_2	u_3	u_4	$\tilde{v}$	$\tilde{w}$	
Q_1	1	1	1	1	2	0	5
Q_2	0	0	0	0	1	1	1
class	P	P	P	P	2P +X	X	

B is not Calabi-Yau: $K_B^{-1} = P + X$

analysis of B with toric methods: topology of O-plane unchanged:

$\chi(D_{\tilde{v}}) + \chi(D_{\tilde{w}}) = \chi(D_v) + \chi(D_w)$

4-fold Y: Weierstrass model $y^2 = x^3 + x\, z^4\, f(u_i, \tilde{v}, \tilde{w}) + z^6\, g(u_i, \tilde{v}, \tilde{w})$

F-Theory uplift

B is not Fano $\leftrightarrow$ generic appearance of singularities!

[Blumenhagen,Grimm,Jurke, TW 0906.0013]

- Type IIB picture:

  D7-tadpole $8([D_v] + [D_w])$

  Naively: $1 \times [8D_v] + 1 \times [8D_w] \leftrightarrow SO(1) \times SO(1)$

  $\Rightarrow \chi^*(Y) = \frac{1}{2}(\chi_o(8\,D_v) + \chi_o(8\,D_w)) + 2\chi(O7) = 1728$

  But: dP_6 along $[D_w]$ is rigid $\Rightarrow$ $\exists$ no single brane of charge $[8D_w]$

  $\Rightarrow$ minimal gauge group: $SO(1) \times SO(8) \leftrightarrow 1 \times [8D_v] + 8 \times [D_w]$

  $\Rightarrow \chi(Y) = 1224$

- F-theory picture:

  Naively: $\chi^*(Y) = 12 \int_B c_1(B)\, c_2(B) + 360 \int_B c_1^3(B) = 1728 \quad \checkmark$

  Taking into account gauge enhancement $SO(8)$ along $D_{\tilde{v}}$:

  $\chi(Y) = \chi^*(Y) - r_{SO(8)}\, c_{SO(8)}\, (c_{SO(8)} + 1) \int_{D_{\tilde{v}}} c_1(D_{\tilde{v}})^2 = 1228 \quad \checkmark$

F-Theory uplift

1) Do general Type IIB configurations survive uplift?

2) Does uplift of IIB geometry allow for interesting non-pert. gauge groups for generic compl. structure?

Gauge enhancements from Tate's algorithm: [Bershadsky et al.'96]

$y^2 + x\,y\,z\,a_1 + y\,z^3\,a_3 = x^3 + x^2\,z^2\,a_2 + x\,z^4\,a_4 + z^6\,a_6,$

$a_i \in H^0(B, K_B^{-i})$

exact gauge group along $D \leftrightarrow$ order of zeroes of a_i and Δ

equiv. to Weierstrass form w/ $f = -3h^2 + \epsilon\eta, g = -2h^3 + \epsilon h\eta - \frac{\epsilon^2}{12}\chi$

$h \leftrightarrow a_1^2 + 4a_2$

general Ansatz: $a_1 = p_1(\mathbf{u})\,\tilde{w}, \qquad a_2 = c_0\,\tilde{v}\,\tilde{w} + p_2(\mathbf{u})\,\tilde{w}^2$

- F-theory with orientifold limit $\leftrightarrow$ choice of compl. structure moduli:
  O7: $h = \tilde{v}\tilde{w} = 0 \Longleftrightarrow p_2(\mathbf{u}) = -\frac{1}{4}p_1^2(\mathbf{u})\,\tilde{w}^2$
- generic choice of $a_1, a_2 \rightarrow$ inherently non-pert. vacua

Tate algorithm

sing. type	discr. $\deg(\Delta)$	group enhancement	coefficient vanishing degrees a_1	a_2	a_3	a_4	a_6
I_0	0	—	0	0	0	0	0
I_1	1	—	0	0	1	1	1
I_2	2	$SU(2)$	0	0	1	1	2
$\mathrm{I}_3^{\mathrm{ns}}$	3	[unconv.]	0	0	2	2	3
$\mathrm{I}_3^{\mathrm{s}}$	3	[unconv.]	0	1	1	2	3
$\mathrm{I}_{2k}^{\mathrm{ns}}$	$2k$	$SP(2k)$	0	0	k	k	$2k$
$\mathrm{I}_{2k}^{\mathrm{s}}$	$2k$	$SU(2k)$	0	1	k	k	$2k$
$\mathrm{I}_{2k+1}^{\mathrm{ns}}$	$2k+1$	[unconv.]	0	0	$k+1$	$k+1$	$2k+1$
$\mathrm{I}_{2k+1}^{\mathrm{s}}$	$2k+1$	$SU(2k+1)$	0	1	k	$k+1$	$2k+1$
II	2	—	1	1	1	1	1
III	3	$SU(2)$	1	1	1	1	2
$\mathrm{IV}^{\mathrm{ns}}$	4	[unconv.]	1	1	1	2	2
IV^{s}	4	$SU(3)$	1	1	1	2	3
$\mathrm{I}_0^{*\,\mathrm{ns}}$	6	G_2	1	1	2	2	3

stolen from: [Bershadsky,Intriligator,Kachru,Morrison,Sadov,Vafa 9605200]

Tate algorithm

sing. type	discr. $\deg(\Delta)$	group enhancement	coefficient vanishing degrees a_1	a_2	a_3	a_4	a_6
$I_0^{*\,ss}$	6	$SO(7)$	1	1	2	2	4
$I_0^{*\,s}$	6	$SO(8)^*$	1	1	2	2	4
$I_1^{*\,ns}$	7	$SO(9)$	1	1	2	3	4
$I_1^{*\,s}$	7	$SO(10)$	1	1	2	3	5
$I_2^{*\,ns}$	8	$SO(11)$	1	1	3	3	5
$I_2^{*\,s}$	8	$SO(12)^*$	1	1	3	3	5
$I_{2k-3}^{*\,ns}$	$2k+3$	$SO(4k+1)$	1	1	k	$k+1$	$2k$
$I_{2k-3}^{*\,s}$	$2k+3$	$SO(4k+2)$	1	1	k	$k+1$	$2k+1$
$I_{2k-2}^{*\,ns}$	$2k+4$	$SO(4k+3)$	1	1	$k+1$	$k+1$	$2k+1$
$I_{2k-2}^{*\,s}$	$2k+4$	$SO(4k+4)^*$	1	1	$k+1$	$k+1$	$2k+1$
$IV^{*\,ns}$	8	F_4	1	2	2	3	4
$IV^{*\,s}$	8	E_6	1	2	2	3	5
III^*	9	E_7	1	2	3	3	5
II^*	10	E_8	1	2	3	4	5

stolen from: [Bershadsky,Intriligator,Kachru,Morrison,Sadov,Vafa 9605200]

F-Theory uplift

- local cancellation of O7-charge

D7 on top of O-plane $\leftrightarrow$ g_s free, no special features in F-theory

- non-local cancellation:

Example: IIB: $8 \times [D_{u_1}] + 16 \times D_w$: $SP(8) \times SO(16)$

F-theory:

$$a_1 = p_1(\mathbf{u})\,\tilde{w}, \quad a_2 = \tilde{v}\,\tilde{w} - \tfrac{1}{4}\big(p_1(\mathbf{u})\,\tilde{w}\big)^2, \quad a_3 = 0,\, a_4 = c\,u_1^4\tilde{w}^4, \quad a_6 = 0$$

Sen limit: $a_3 \to \epsilon a_3, \quad a_4 \to \epsilon a_4, \quad a_6 \to \epsilon^2 a_6$

$\Delta_F \simeq \epsilon^2 u_1^8\,\tilde{w}^{10}\,(\tilde{v}^2 - \epsilon 4 c u_1^4 \tilde{w}^2) \Longrightarrow \Delta_{IIB} \simeq \epsilon^2\,u_1^8\,\tilde{w}^{10}\,\tilde{v}^2$

$\rightarrow$ non-pert. splitting on part of the O-plane along $D_{\tilde{v}}$ in F-theory

- minimal gauge group

IIB: $SO(8)$ from $1 \times [8D_w] + 8 \times [D_v]$

F-theory : all $a_n = \tilde{w}^{d_n}(\ldots)$ with $(d_1, d_2, d_3, d_4, d_6) = (1, 1, 2, 2, 3)$

$\Delta_F = \tilde{w}^6 \Rightarrow G_2$ Sen limit: discard $a_6/a_i \to 0 \Rightarrow G_2 \to SO(8)$

F-Theory uplift

Away from IIB sublocus: also exceptional gauge enhancements possible

Example: E_6 along dP_6 $D_{\tilde{w}}$:

$$a_1 = p_{(1,0)}\,\tilde{w}, \quad a_2 = p_{(2,0)}\,\tilde{w}^2, \quad a_3 = p_{(3,1)}\tilde{w}^2, \quad a_4 = p_{(4,1)}\,\tilde{w}^3$$
$$a_6 = p_{(6,1)}\,\tilde{w}^5$$

further enhancements: E_7 on $\tilde{w} = 0 = p_{3,1} \leftrightarrow$ **27** matter

E_8: $\tilde{w} = 0 = p_{3,1} = p_{4,1}$: $\leftrightarrow$ $\mathbf{27}^3$ Yukawa

$\Rightarrow$ smooth deformations relating perturbative description with non-pert. regime!

$\Rightarrow$ **Can we deform SU(5) GUT model so as produce $\mathbf{10}\,\mathbf{10}\,\mathbf{5_H}$ while keeping global consistency of IIB setup?**

F-Theory uplift

Example of lifting of exchange involutions: Double del Pezzo transition

	u_1	u_2	u_3	v_1	v_2	w_1	w_2	
Q_1	1	1	1	1	1	0	0	5
Q_2	0	0	0	0	1	1	0	2
Q_3	0	0	0	1	0	0	1	2
class	H	H	H	H+Y	H+X	X	Y	

Involution: $\sigma : v_1 \leftrightarrow v_2, \qquad w_1 \leftrightarrow w_2 \Rightarrow$ O7-plane: $v_1 w_1 - v_2 w_2 = 0$

map: $(u_i, v_1, v_2, w_1, w_2) \mapsto (u_i, v_1\, v_2, w_1 w_2, v_1 w_1 + v_2 w_2) \equiv (u_i, v, h, w)$

	u_1	u_2	u_3	v	h	w	
Q_1	1	1	1	2	1	0	5
Q_2	0	0	0	1	1	1	2
class	P	P	P	2P+X	P+X	X	

F-Theory uplift

Extensions to 2 more del Pezzo's (inert under σ) $\rightarrow Q^{dP_9^4}$ 3-gen. example!

- non-Fano property of base $\leftrightarrow$ singular curve on O7 (self-intersection)
- match of D3-tadpole modulo singularities
- non-pert. enhancement away from IIB limit

SU(5) GUTs (away from IIB limit):

$$a_1 = p_{(1,1)},\ a_2 = p_{(2,1)}\, w,\ a_3 = p_{(3,1)}, w^2,\ a_4 = p_{(4,1)} w^3,\ a_6 = p_{(5,0)}\, w^6$$

Matter curves:

- $\mathbf{10} \leftrightarrow SO(10) : \{w = p_{(1,1)} = 0\}$
- $\overline{\mathbf{5}} \leftrightarrow SU(6) : \{w = p_{(3,1)} = 0\}$

Yukawa Couplings:

- $\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}}_{\mathbf{H}} \leftrightarrow SO(12)$: $\{w = p_{(1,1)} = p_{(3,1)} = 0\}$

  # points: $X(P + X)(3P + X) = 1$ ✓

- $\mathbf{10}\,\mathbf{10}\,\mathbf{5}_{\mathbf{H}} \leftrightarrow E_6$: $\{w = p_{(1,1)} = p_{(2,1)} = 0\}$

  # points: $X(P + X)(2P + X) = 0$

Conclusions

Type IIB orientifolds suitable arena for SU(5) GUT model building

Recent technological input:

- GUT breaking by $U(1)_Y$ flux à la BHV/DW
- $\mathbf{10}\,\mathbf{10}\,\mathbf{5_H}$ couplings in Type IIB by exotic D-brane instantons

Realisation of many, but not all phenomenologically desirable features in globally consistent Type IIB orientifolds

Uplift of geometries to F-theory achieved in some examples

- presence of del Pezzo divisors useful for F-theory GUTs
- smooth deformation of complex structure away from IIB limit generates non-pert. gauge groups

Aim: Find example where Yukawas generated in this manner

Open challenge: Understand implementation of gauge flux via G_4 in F-theory + resulting global constraints