New SCFTs from Wrapped Branes

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Motivation

- Large classes of interacting p-dimensional superconformal field theories (SCFTs) obtained from a d-dimensional theory "compactified" on a manifold $\mathcal{M}_q$ of dimensions $q = d - p$. [Vafa-Witten], [Bershadsky-Johansen-Sadov-Vafa], [Witten], [Klemm-Lerche-Mayr-Vafa-Warner], [Maldacena-Núñez], [Kapustin], [Gaiotto], [Gaiotto-Moore-Neitzke], [Dimofte-Gukov-Gaiotto], [Cecotti-Cordova-Vafa], ...
- Strings and branes provide a useful unifying picture. The p-dimensional theory is the low-energy limit of M/D-branes wrapped on $\mathcal{M}_q$. Some supersymmetry is preserved by a partial topological twist - naturally incorporated by the brane construction. [Bershadsky-Sadov-Vafa]
- These constructions lead to interesting "dualities" between the p-dimensional SCFT and a (topological) theory on $\mathcal{M}_q$. [AGT], [Gadde-Pomoni-Rastelli-Razamat], [DGG], [Cecotti-Cordova-Vafa], ...
- The p-dimensional SCFTs typically admit a large N limit and have holographic duals which can be explicitly constructed. New examples of AdS/CFT.

Overview

- Here I will focus on $d = 6$ ((2,0) theory), $d = 4$ ($\mathcal{N} = 4$ SYM), $q = 2$ (Riemann surfaces), and $q = 4$ (four-manifolds).
- New class of 4D $\mathcal{N} = 1$ SCFTs obtained from the $(2,0)$ theory on a Riemann surface.
- A plethora of new 2D $(0,2)$ SCFTs from $\mathcal{N} = 4$ SYM on a Riemann surface and the $(2,0)$ theory on a four-manifold.
- Proof of c-extremization - a general principle in 2D $(0,2)$ SCFTs which determines the exact superconformal R-symmetry. Analogous to a-maximization in 4D and F-maximization in 3D. [Intriligator-Wecht], [Jafferis], [Closset-Dumitrescu-Festuccia-Komargodski-Seiberg]
- All these SCFTs have explicit holographic duals. Large classes of novel supersymmetric AdS_3 and AdS_5 (warped) compactifications of string/M-theory.
- Use non-perturbative tools - anomalies, a-maximizaton, c-extremization, holography.

Disclaimer: Here I always take $\mathcal{M}_q$ to be compact of constant curvature for $q = 2$ and compact and Einstein for $q = 4$. Generalizations are possible and very interesting! [Anderson-Beem-NB-Rastelli], [Gaiotto-Maldacena]

Basic idea

A supersymmetric field theory on a generic curved manifold is no longer supersymmetric. To preserve supersymmetry perform a "topological twist", i.e. use the R-symmetry to cancel the space-time curvature [Witten]

$$A_\mu = -\frac{1}{4}\omega_\mu \; , \qquad \rightarrow \qquad \widetilde{\nabla}_\mu \epsilon = \left(\partial_\mu + \frac{1}{4}\omega_\mu + A_\mu\right)\epsilon = \partial_\mu \epsilon = 0 \; .$$

Branes in string/M theory wrapping curved cycles preserve supersymmetry in this way [Bershadsky-Sadov-Vafa], [Maldacena-Núñez] .

In addition one is free to turn on background gauge fields for "flavor" symmetries.

Why a CFT at low energies? Roughly the same physics as charged particles in external magnetic field - Landau levels. The CFT describes the lowest Landau level.

4D $\mathcal{N} = 1$ SCFTs

Twists and branes

Put the 6D A_{N-1} $(2,0)$ theory, i.e. N M5-branes, on a Riemann surface $\Sigma_{\mathfrak{g}}$. The supercharges decompose under

$$SO(1,3) \times SO(2)_{\Sigma_{\mathfrak{g}}} \times U(1)_1 \times U(1)_2 \subset SO(1,5) \times SO(5)_R \ .$$

Define

$$SO(2)' = SO(2)_{\Sigma_{\mathfrak{g}}} + \frac{\ell_1}{2\mathfrak{g}-2} U(1)_1 + \frac{\ell_2}{2\mathfrak{g}-2} U(1)_2 \ .$$

For $\ell_1 + \ell_2 = 2\mathfrak{g} - 2$ there are 4 invariant supercharges, i.e. $\mathcal{N} = 1$ supersymmetry in 4D.

Realized in M-theory by N M5-branes wrapping a holomorphic 2-cycle in a CY_3 which is a sum of complex line bundles over $\Sigma_{\mathfrak{g}}$

$$\mathcal{L}_1 \oplus \mathcal{L}_2 \to \Sigma_{\mathfrak{g}} \ .$$

The Calabi-Yau condition is $\ell_1 + \ell_2 = 2\mathfrak{g} - 2$.

Special cases

- For $\ell_1 = 0$ (or $\ell_2 = 0$) the local geometry is $T^*(\Sigma_{\mathfrak{g}})$ and there is an enhancement to 8 supercharges i.e. $\mathcal{N} = 2$ supersymmetry in 4D. [Maldacena-Núñez], [Gaiotto]
- For $\ell_1 = \ell_2 = \mathfrak{g} - 1$ the local geometry is $K^{1/2} \oplus K^{1/2} \to \Sigma_{\mathfrak{g}}$. [Maldacena-Núñez], [Benini-Tachikawa-Wecht]

Anomalies

Use the M5-brane anomaly to compute the central charges of the 4D theories. For $(2,0)$ theory of type $G = A_N, D_N, E_{6,7,8}$ the anomaly is [Witten], [Harvey-Minasian-Moore], [Yi], [Intriligator]

$$I_8[G] = \frac{r_G}{48}\left[p_2(N) - p_2(T) + \frac{1}{4}(p_1(T) - p_1(N))^2\right] + \frac{d_G h_G}{24} p_2(N) \ .$$

The $\mathcal{N} = 1$ twist is implemented geometrically by

$$n_1 = -\frac{1+z}{2} t + (1+\epsilon) c_1(F) \ , \qquad n_2 = -\frac{1-z}{2} t + (1-\epsilon) c_1(F) \ ,$$

here n_i and t are the Chern roots of $\mathcal{L}_i$ and Σ_g. The rational parameter specifying the twist is $z = (\ell_1 - \ell_2)/(2\mathfrak{g} - 2)$. The 4D R-symmetry is determined by ϵ.

Integrate $I_8[G]$ over $\Sigma_{\mathfrak{g}}$ to get the 4D anomaly polynomial from which one reads off the R-anomalies

$$I_6 = \frac{\mathrm{tr} R^3}{6} c_1(F)^3 - \frac{\mathrm{tr} R}{24} c_1(F) p_1(T_4) \ .$$

Central charges

Use the R-anomaly to find the a and c central charges as a function of ϵ [Anselmi-Freedman-Grisaru-Johansen]

$$a(\epsilon) = \frac{9}{32}\mathrm{tr}R^3 - \frac{3}{32}\mathrm{tr}R \,, \qquad c(\epsilon) = \frac{9}{32}\mathrm{tr}R^3 - \frac{5}{32}\mathrm{tr}R \,.$$

Use a-maximization [Intriligator-Wecht] to fix

$$\epsilon = \frac{\eta - \zeta}{3(1+\eta)z} \,,$$

and find

$$a = (\mathfrak{g}-1)r_G \frac{\zeta^3 - \eta^3 + (1+\eta)(9+21\eta+9\eta^2)z^2}{48(1+\eta)^2 z^2} \,,$$

$$c = (\mathfrak{g}-1)r_G \frac{\zeta^3 - \eta^3 + (1+\eta)(6+\zeta+17\eta+9\eta^2)z^2}{48(1+\eta)^2 z^2} \,,$$

where

$$\eta \equiv h_G(1+h_G) \,, \qquad \zeta \equiv \sqrt{\eta^2 + (1+4\eta+3\eta^2)z^2} \,.$$

I assumed $\mathfrak{g} > 1$ but similar results hold for S^2 and T^2. Note that $a > 0$ and $c > 0$.

For $SU(N)$ $r_G = N-1$ and $h_G = N$.

Central charges

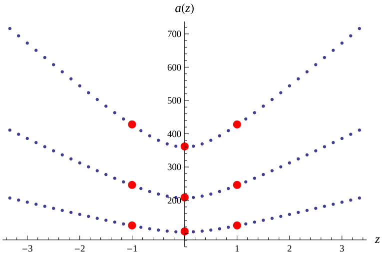
At large N the central charges are

$$a = c = \frac{(1+3z^2)^{3/2} + 9z^2 - 1}{48z^2}(\mathfrak{g}-1)N^3 \ ,$$

and the R-symmetry is determined by

$$\epsilon = \frac{1-\sqrt{1+3z^2}}{3z} \ .$$

An example: $a(z)$ for $G = A_{N-1}$, $\mathfrak{g} = 7$ and $N = 4, 5, 6$. The Maldacena-Núñez theories are at $z = 0$ and $z = \pm 1$ and there are $\mathfrak{g} - 2$ SCFTs in between.



Supergravity setup

To study the twisted theory holographically we need only modes that lie within the maximal 7D gauged supergravity. [Pernici-Pilch-van Nieuwenhuizen], [Maldacena-Núñez] This is a consistent truncation of 11D supergravity on S^4. [Vaman-Nastase-van Nieuwenhuizen]

Strategy:

- Identify the relevant fields of the maximal 7D gauged supergravity and a suitable Ansatz.
- Derive a system of BPS equations and solve them to find $AdS_5 \times \Sigma_{\mathfrak{g}}$ vacua.
- Construct explicit holographic RG flows from an asymptotically locally AdS_7 space to the AdS_5 fixed points $\rightarrow$ the SCFTs are dynamically realized.
- Uplift the solutions to 11D using standard formulæ. [Cvetič et. al.]
- Study the solutions and calculate various quantities in the dual field theory.
- Convert to "canonical coordinates" to identify the superconformal R-symmetry. [Gauntlett-Martelli-Sparks-Waldram]

Solutions in 11D

The solutions of 11D supergravity are of the form $AdS_5 \times_w M_6$ with M_6 an S^4 fibered over $\Sigma_{\mathfrak{g}}$

$$ds_{11}^2 = \Delta^{1/3}\left[e^{2f}\frac{(-dt^2+dz_1^2+dz_2^2+dz_3^2+dr^2)}{r^2}+e^{2g}\frac{(dx^2+dy^2)}{y^2}\right]$$

$$+\frac{1}{4}\Delta^{-2/3}\left[X_0^{-1}d\mu_0^2+\sum_{i=1}^{2}X_i^{-1}\left(d\mu_i^2+\mu_i^2\left(d\phi_i+A^{(i)}\right)^2\right)\right] \ ,$$

where $X_0=(X_1X_2)^{-2}$ and

$$\Delta=\sum_{a=0}^{2}X_a\mu_a^2 \ , \qquad \mu_0=\cos\alpha \ , \qquad \mu_1=\sin\alpha\cos\theta \ , \qquad \mu_2=\sin\alpha\sin\theta \ ,$$

$$F^{(1)}\equiv dA^{(1)}=\frac{1+z}{8}\frac{dx\wedge dy}{y^2} \ , \qquad F_{xy}^{(2)}\equiv dA^{(2)}=\frac{1-z}{8}\frac{dx\wedge dy}{y^2} \ .$$

The Riemann surface is a quotient of $\mathbb{H}_2$ by $\Gamma\in PSL(2,\mathbb{R})$. Can treat also S^2 and T^2 with the same approach.

Solutions in 11D

The solutions are then determined by

$$
\begin{aligned}
(X_1)^5 &= \frac{1+7\,z+7\,z^2+33\,z^3-(1+4\,z+19\,z^2)\sqrt{1+3\,z^2}}{4\,z(1-z)^2}\ , \\
X_1(X_2)^{-1} &= \frac{1+z}{2\,z+\sqrt{1+3\,z^2}}\ , \\
e^f &= (X_1X_2)^2\ , \\
e^{2g} &= \frac{X_1X_2}{8}\,((1-z)\,X_1+(1+z)\,X_2)\ .
\end{aligned}
$$

A family of $\frac{1}{4}$-BPS AdS_5 vacua labeled by the discrete parameters $\mathfrak{g}$ and z!

For $z=\pm 1$ supersymmetry is enhanced and one finds the $\mathcal{N}=2$ Maldacena-Núñez solution. The $\mathcal{N}=1$ Maldacena-Núñez solution is at $z=0$ and has $SU(2)_F\times U(1)_R$ symmetry.

Properties of the dual SCFT

- The central charge in the large N limit is

$$a = c = \frac{8(\mathfrak{g}-1)}{3} e^{2g+3f} N^3 = \frac{(1+3\,z^2)^{3/2} + 9z^2 - 1}{48\,z^2} (\mathfrak{g}-1) N^3 \ .$$

  This is the same as the large N limit of the result from anomalies.

- The dual SCFT has a $U(1)$ superconfromal R-symmetry and a $U(1)$ flavor symmetry realized in the gravity solutions by the two Killing vectors

$$\partial_\psi = (1+\epsilon)\partial_{\phi_1} + (1-\epsilon)\partial_{\phi_2} \ , \qquad \partial_\chi = \partial_{\phi_1} - \partial_{\phi_2} \ .$$

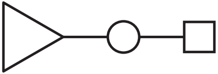
  The same ϵ as obtained by a-maximization!

- The $\mathcal{N}=1$ conformal manifold is the product of the complex structure moduli space and the space of flat $U(1)$ connections on $\Sigma_{\mathfrak{g}}$. The complex dimension of this space is $4\mathfrak{g}-3$. It can be very large!

T_N

T_N are isolated strongly coupled 4D $\mathcal{N} = 2$ SCFTs obtained by wrapping N M5-branes on S^2 with three punctures. They have $SU(2) \times U(1)$ R-symmetry and $SU(N)^3$ flavor symmetry. [Nemeschansky-Minahan], [Argyres-Seiberg], [Gaiotto], [Gaiotto-Maldacena]

They can be viewed as new building blocks for constructing $\mathcal{N} = 2$ quiver gauge theories.



There is no known Langrangian description of T_N theories for $N > 2$, however there is quite a bit known about them.

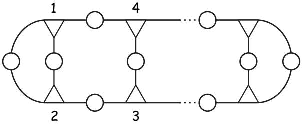
The central charges of a T_N theory are [Gaiotto-Maldacena]

$$a_{T_N} = \frac{N^3}{6} - \frac{5N^2}{16} - \frac{N}{16} + \frac{5}{24} , \qquad c_{T_N} = \frac{N^3}{6} - \frac{N^2}{4} - \frac{N}{12} + \frac{1}{6} .$$

Generalized $\mathcal{N} = 2$ quivers

Build generalized $\mathcal{N} = 2$ quiver theories by "gluing" T_N theories, i.e. gauge the diagonal $SU(N)$ of two flavor groups with an $\mathcal{N} = 2$ vector multiplet. [Gaiotto]

Build closed quivers (no $SU(N)$ flavor symmetries) by following the rules for gluing spheres with three punctures to construct a compact Riemann surface. Such a quiver is dual to the $\mathcal{N} = 2$ Maldacena-Núñez solutions ($z = \pm 1$) and corresponds to M5-branes wrapped on a compact Riemann surface. [Gaiotto], [Gaiotto-Maldacena]



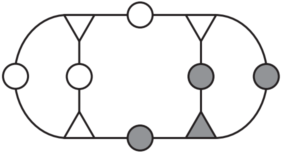
For genus $\mathfrak{g} > 1$ surface we have $2\mathfrak{g} - 2$ T_N blocks and $3\mathfrak{g} - 3$ $\mathcal{N} = 2$ vector multiplets.

S-dual theories are realized by different quivers that give a Riemann surface of the same genus.

Generalized $\mathcal{N} = 1$ quivers

There is a more general gluing procedure! Connect T_N theories by either an $\mathcal{N} = 1$ or $\mathcal{N} = 2$ vector multiplets and flow to the IR.

Label each T_N by a sign $\sigma = \pm 1$ and build closed generalized $\mathcal{N} = 1$ quivers. Each quiver is labeled by two positive integers (ℓ_1, ℓ_2), with $\ell_1 + \ell_2 = 2\mathfrak{g} - 2$, which correspond to the number of $+$ and $-$ signs respectively. Same as the degrees of the line bundles in the geometric picture.



This is an example with $\mathfrak{g} = 3$, $\ell_1 = 1$, $\ell_2 = 3$.

These generalized quiver theories flow in the IR to the $\mathcal{N} = 1$ SCFTs with $-1 \leq z \leq 1$. The $\mathcal{N} = 2$ and $\mathcal{N} = 1$ Maldacena-Núñez theories have $\ell_1 = 0$ (or $\ell_2 = 0$) and $\ell_1 = \ell_2 = \mathfrak{g} - 1$ respectively.

Generalized $\mathcal{N} = 1$ quivers

- The resulting a and c central charges match with the ones computed via the M5-brane anomaly polynomial and supergravity.
- The dimension of the conformal manifold is $4\mathfrak{g} - 3$ and matches with the supergravity result.
- We can compute the dimension of some of the protected operators in the theory.
- $\mathcal{N} = 1$ dualities - different UV realization of a quiver with the same ℓ_1 and ℓ_2 flow to the same IR fixed point.
- Tests: central charges + R-symmetry + superconformal index + map to "old" $\mathcal{N} = 1$ dualities. [Beem-Gadde], [Gadde-Maruyoshi-Tachikawa-Yan], [Csaki-Schmaltz-Skiba-Terning]

2D $(0,2)$ SCFTs

c-extremization

Consider a 2D local relativistic QFT with $U(1)^n$ Abelian global symmetry group. There are "gauge" and gravitational anomalies [Alvarez-Gaume-Witten]

$$\nabla^\mu J^I_\mu = \sum_L \frac{k^{IL}}{8\pi} F^L_{\mu\nu}\varepsilon^{\mu\nu} \, , \qquad \nabla_\mu T^{\mu\nu} = \frac{k}{96\pi} g^{\nu\alpha}\varepsilon^{\mu\rho}\partial_\mu\partial_\beta\Gamma^\beta_{\alpha\rho} \, .$$

The symmetric tensor k^{IL} and the constant k are the 't Hooft anomaly coefficients

$$k^{IL} = \mathrm{Tr}_{\text{“Weyl”}}\gamma^3 Q^I Q^L \, , \qquad k = \mathrm{Tr}_{\text{“Weyl”}}\gamma^3 \, .$$

When the theory is conformal k^{IL} and k are related to central terms in the Virasoro and current algebras.

In a 2D SCFTs with $(0,2)$ supersymmetry there is a special conserved current, Ω_μ, in the multiplet of the energy momentum tensor - the R-current

$$k^{RR} = \frac{c_r}{3} \, , \qquad k = c_r - c_\ell \, .$$

Find this current using only anomalies without detailed knowledge of the IR CFT?

Important assumptions - The CFT is unitary and it has a normalizable vacuum!

c-extremization

Consider a trial R-current Ω^{tr}_μ

$$\Omega^{\text{tr}}_\mu(t) = \Omega_\mu + \sum_{I\,(\neq R)} t_I J^I_\mu \ .$$

Then construct a trial central charge $c^{\text{tr}}_r(t)$ proportional to the anomaly of the trial R-symmetry:

$$c^{\text{tr}}_r(t) = 3\left(k^{RR} + 2\sum_{I\neq R} t^I k^{RI} + \sum_{I,L\neq R} t^I t^L k^{IL}\right) \ .$$

One can prove that in a renormalization scheme in which all currents are primary fields

$$k^{RI} = 0\ , \qquad\qquad \forall\, I \neq R\ .$$

This is an extremality condition for $c^{\text{tr}}_R(t)$.

$$\frac{\partial c^{\text{tr}}_r(t^*)}{\partial t^I} = 0\ , \quad \forall\, I \neq R\ , \quad \rightarrow \quad c^{\text{tr}}_r(t^*) = c_r\ .$$

Similar to a-maximization in 4D and F-maximization in 3D. [Intriligator-Wecht], [Jafferis], [Closset-Dumitrescu-Festuccia-Komargodski-Seiberg]

Caveat - May fail if there are accidental global symmetries in the IR!

D3-branes on Riemann surfaces

Study $\mathcal{N}=4$ SYM with gauge group G on a Riemann surface $\Sigma_{\mathfrak{g}}$ with a partial topological twist. [Bershadsky-Johansen-Sadov-Vafa], [Maldacena-Núñez]

Turn on an $SO(2)^3 \in SO(6)_R$ background gauge field

$$F = -T dvol_{\Sigma_{\mathfrak{g}}} \ , \qquad T = a_1 T_1 + a_2 T_2 + a_3 T_3 \ .$$

To preserve $(0,2)$ supersymmetry impose

$$a_1 + a_2 + a_3 = -\kappa \ , \qquad \kappa = \begin{cases} +1 & S^2 \\ 0 & T^2 \\ -1 & H^2 \end{cases}$$

This construction is realized in string theory by N D3-branes wrapping a holomorphic 2-cycle in a CY_4 which is a sum of complex line bundles with degrees ℓ_i over $\Sigma_{\mathfrak{g}}$

$$\mathcal{L}_1 \oplus \mathcal{L}_2 \oplus \mathcal{L}_3 \to \Sigma_g \ .$$

With $\ell_i = -2\kappa(\mathfrak{g}-1)a_i$, for $\mathfrak{g}=1$, $\ell_i = a_i$.

Anomalies and c-extremization are sufficient to argue that the 2D theory is conformal for a wide range of parameters.

D3-branes on Riemann surfaces

The 2D theory inherits the $SO(2)^3$ symmetry. The trial R-symmetry is

$$T_R = \epsilon_1 T_1 + \epsilon_2 T_2 + (2 - \epsilon_1 - \epsilon_2) T_3 \ .$$

The number of 2D massless fermions is (index theorem)

$$n_R^{(\sigma)} - n_L^{(\sigma)} = \frac{1}{2\pi} \int_{\Sigma_\mathfrak{g}} \mathrm{Tr}_\sigma F \ , \qquad c_r^{tr}(\epsilon) = 3 d_G \sum_\sigma (n_R^{(\sigma)} - n_L^{(\sigma)})(q_R^{(\sigma)}(\epsilon))^2 \ .$$

Here $\sigma \in \mathbf{4}$ of $SU(4)_R$ and labels the 4D gaugini.

Extremize with respect to ϵ_i to find

$$c_r = 12 \eta_\Sigma d_G \frac{a_1 a_2 a_3}{(a_1 + a_2 + a_3)^2 - 2(a_1^2 + a_2^2 + a_3^2)} \ , \qquad \eta_\Sigma = \begin{cases} 2|\mathfrak{g} - 1| & \mathfrak{g} \neq 1 \\ 1 & \mathfrak{g} = 1 \end{cases}$$

Note that $c_r - c_\ell = 0$, i.e. no gravitational anomaly.

c-extremization is essential to find the correct result. Without it one finds a mismatch between the supergravity and field theory result for the central charges. [Almuhairi-Polchinski]

Supergravity solutions

The IIB metric is (there is only 5-form flux)

$$ds_{10}^2 = \Delta^{1/2}\Big[e^{2f}\,\frac{-dt^2+dz^2+dr^2}{r^2} + e^{2g}\,ds^2_{\Sigma_\mathfrak{g}}\Big] + \Delta^{-1/2}\sum\nolimits_i (X_i)^{-1}\left(d\mu_i^2 + \mu_i^2\left(d\varphi_i + A^{(i)}\right)^2\right) \,,$$

where

$$\Delta = \sum_i X_i\mu_i^2 \,, \qquad X_1X_2X_3 = 1 \,, \qquad \sum_i \mu_i^2 = 1 \,,$$

The parameters specifying the solution are

$$e^{2g} = \frac{a_1X_2 + a_2X_1}{2} \,, \qquad (X_1)^2X_2 = \frac{a_1(a_2+a_3-a_1)}{a_3(a_1+a_2-a_3)} \,,$$

$$e^{f} = \frac{2}{X_1+X_2+X_3} \,, \qquad X_1(X_2)^2 = \frac{a_2(a_1+a_3-a_2)}{a_3(a_1+a_2-a_3)} \,.$$

There is a good AdS_3 vacuum only in some region in the (a_1, a_2) parameter space.

Holographic RG flows from an asymptotically locally AdS_5 space to these AdS_3 fixed points.

Salient features

- For $a_1 = a_2 = 0$ and $a_3 = 1$ supersymmetry is enhanced to $(4,4)$. The SCFT is a σ-model on the Hitchin moduli space on $\Sigma_{\mathfrak{g}}$. [Bershadsky-Johansen-Sadov-Vafa]

- For $a_1 = a_2 = \frac{1}{2}$ and $a_3 = 0$ we have $(2,2)$ supersymmetry and

$$c_r = c_\ell = 3(\mathfrak{g} - 1)d_G \ .$$

  Is this a σ-model with a CY target?

- Find the supergravity duals of these SCFTs by backreacting the D3-branes. The resulting solutions are new smooth supersymmetric vacua of IIB supergravity of the form $AdS_3 \times_w M_7$.

- The central charge computed in supergravity matches with the one in field theory. The superconformal R-symmetry is determined by a Killing vector constructed as a bilinear in the Killing spinor and also agrees with c-extremization.

- The manifolds M_7 are S^5 bundles over $\Sigma_{\mathfrak{g}}$ and are a lot like Sasaki-Einstein manifolds. [Gauntlett-Kim]

M5-branes on four-manifolds

Study M5-branes (i.e. 6D (2,0) theory) wrapping calibrated 4-cycles, $\mathcal{M}_4$, in special holonomy manifolds.

- $\Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2}$ in $CY_4 \to$ (at least) $(0,2)$ SCFTs. Generalized 4D quiver SCFTs on a Riemann surface. Dual supergravity solutions exist when either $\mathfrak{g}_1 > 1$ or $\mathfrak{g}_2 > 1$.
- Kähler 4-cycle in $CY_4 \to (0,2)$ SCFTs. [Ganor] Dual supergravity solution exists for negatively curved Kähler-Einstein metric on $\mathcal{M}_4$. One parameter family of SCFTs.
- Kähler SLAG 4-cycle in $HK_2 \to (1,2)$ SCFTs. Dual supergravity solution exists for $\mathcal{M}_4 = \mathbb{CH}^2/\Gamma$. [Gauntlett-Kim]
- Co-associative 4-cycle in $G_2 \to (0,2)$ SCFTs. Dual supergravity solution exists for $\mathcal{M}_4$ Einstein with ASD Weyl tensor. [Gauntlett-Kim-Waldram]

These are all twists with at least $(0,2)$ supersymmetry. Other constructions with less supersymmetry are possible. [Gauntlett-Kim-Waldram]

M5-branes on four-manifolds

- Integrate the M5-brane anomaly polynomial I_8 over $\mathcal{M}_4$ to find the anomaly polynomial in 2D
$$I_4 = \frac{c_r}{6} c_1(F)^2 - \frac{c_r - c_\ell}{24} p_1(T_2) ,$$
to calculate the central charges of these SCFTs. They scale as N^3. Again, c-extremization is essential.
- In all cases one can find AdS_3 supergravity duals of these field theories. Many new examples of AdS_3/CFT_2 in M-theory. The supergravity central charges match with the ones computed by anomalies and c-extremization.
- Typically the SCFTs are "chiral", i.e. $c_r - c_\ell \neq 0$. Well-known for 2d CFTs coming from M5-branes. [Ganor], [Maldacena-Strominger-Witten], [Kraus-Larsen], ...
- When $CY_4 = T^*(\Sigma_{\mathfrak{g}_1}) \times T^*(\Sigma_{\mathfrak{g}_2})$ we find $(2,2)$ SCFTs with $(\mathfrak{g}_i > 1)$
$$c_r = c_\ell = (\mathfrak{g}_1 - 1)(\mathfrak{g}_2 - 1)(4 d_G h_G + 3 r_G) .$$
This is an integer multiple of 3 for any simply laced G! Is this a σ-model on a CY target?
- A 2D/4D correspondence à la AGT. Hints from the work of Vafa-Witten on $\mathcal{N} = 4$ SYM on $\mathcal{M}_4$, i.e. $(2,0)$ theory on $T^2 \times \mathcal{M}_4$. [Gadde-Gukov-Putrov]

Summary

We made some progress

- An infinite class of new 4D $\mathcal{N} = 1$ SCFTs arising from N M5-branes wrapped on Riemann surfaces. Gravity duals of these SCFTs at large N.
- Several checks of the correspondence - central charges, R-symmetry, dimensions of chiral primary operators, dimension of the conformal manifold. New $\mathcal{N} = 1$ dualities.
- Proof of c-extremization for 2D $(0, 2)$ SCFTs.
- Evidence for new 2D $(0, 2)$ SCFTs, with explicit holographic duals, arising from wrapped D3-branes and M5-branes. Examples of the utility of c-extremization.
- Novel supersymmetric AdS_3 and AdS_5 vacua of IIB and 11D supergravity.

Outlook

... but there are still many things to understand

- What are the field theories dual to the $AdS_5 \times S^2$ and $AdS_5 \times T^2$ solutions as well as the ones dual to the $AdS_5 \times \Sigma_g$ solutions with $|z| > 1$? RG flows between the $\mathcal{N} = 1$ theories? Moduli space? $\mathcal{N} = 1$ curve?...
- $\mathcal{N} = 1$ 4D/2D correspondence à la AGT? [Beem-Gadde]
- Is there a genuinely $\mathcal{N} = 1$ analog of the T_N theories? Understand punctures on the Riemann surface, i.e. flavor symmetries in the field theory. [Bah]
- Construct directly the two-dimensional $(0, 2)$ SCFTs. Should be feasible for the ones coming from D3-branes. [Kapustin]
- Study "standard" 4D $\mathcal{N} = 1$ quiver SCFTs on $\Sigma_{\mathfrak{g}}$. [in progress]
- Other applications of c-extremization (maybe in $(0, 2)$ GLSMs)? Relation to c-theorem?
- Gravity dual of c-extremization? Works in 3d gauged supergravity[Karndumri-Ó Colgáin]. Volume minimization? [Martelli-Sparks-Yau]
- Extremization principle in superconformal quantum mechanics? Application to M2-branes on a Riemann surface. Applications to BPS black holes? [in progress]

THANK YOU!