Quark-Gluon Plasma and Heavy Ion Collisions

II – Collective effects, Hydrodynamics, Phenomenology

François Gelis

CEA/Saclay
General outline

I : Physics of the QGP, Field theory at finite $T$

II : Collective effects, Hydrodynamics, Phenomenology
Collective phenomena

Relativistic hydrodynamics

Phenomenology
Collective phenomena in the QGP
Collective phenomena

- Phenomena involving many elementary constituents
- Long wavelength compared to the typical distance between constituents
- Small frequency or energy

Major collective phenomena:
- Quasi-particles
- Debye screening
- Landau damping
- Collisional width
In order to assess how the medium affects the propagation of excitations, one must compute the photon (gluon) polarization tensor $\Pi^{\mu\nu}(x, y) \equiv \langle J^\mu(x)J^\nu(y) \rangle$.

The photon (or gluon for QCD) self-energy can be resummed on the propagator. Diagrammatically, this amounts to summing:

\[ + \quad + \quad + \quad + \quad + \quad + \ldots \]

The properties of the medium can be read off the analytic properties of this resummed propagator (cuts, poles, ...).
Reminder: the photon polarization tensor $\Pi^{\mu\nu}$ is transverse. At $T = 0$, this implies:

$$\Pi^{\mu\nu}(P) = \left( g^{\mu\nu} - \frac{P^\mu P^\nu}{P^2} \right) \Pi(P^2)$$

- this is due to gauge invariance and Lorentz invariance
- Exercise: this property ensures that the photon remains massless at all orders of perturbation theory

This formula is not valid at $T > 0$, because there is a preferred frame (in which the plasma velocity is zero)

- the tensorial decomposition of $\Pi^{\mu\nu}$ is more complicated, and the photon acquires an effective mass
Dressed propagator

- At finite $T$, the tensorial decomposition of $\Pi^{\mu\nu}$ is:

$$\Pi^{\mu\nu}(P) = P_T^{\mu\nu}(P) \Pi_T(P) + P_L^{\mu\nu}(P) \Pi_L(P)$$

with the following projectors (in the plasma rest frame)

$$P_{ij}^T(P) = g_{ij} + \frac{p^i p^j}{\vec{p}^2}, \quad P_{0i}^T(P) = 0, \quad P_{00}^T(P) = 0$$

$$P_{ij}^L(P) = -\frac{(p^0)^2 p^i p^j}{\vec{p}^2 P^2}, \quad P_{0i}^L(P) = -\frac{p^0 p^i}{P^2}, \quad P_{00}^L(P) = -\frac{\vec{p}^2}{P^2}$$

- Therefore, we have

$$\Pi^{\mu\mu}(P) = 2\Pi_T(P) + \Pi_L(P), \quad \Pi^{00}(P) = -\frac{\vec{p}^2}{P^2} \Pi_L(P)$$

- This leads to the following propagator:

$$D^{\mu\nu}(P) = P_T^{\mu\nu}(P) \frac{1}{P^2 - \Pi_T(P)} + P_L^{\mu\nu}(P) \frac{1}{P^2 - \Pi_L(P)}$$
Check the following properties of the tensors $P_{T,L}^{\mu \nu}$:

$$P_T^{\mu} = 2$$

$$P_L^{\mu} = 1$$

$$P_T^{\mu} P_T^{\alpha \nu} = P_T^{\mu \nu}$$

$$P_L^{\mu} P_L^{\alpha \nu} = P_L^{\mu \nu}$$

$$P_T^{\mu} P_L^{\alpha \nu} = 0$$
Dressed propagator

- The calculation of $\Pi^\mu_\mu$ and $\Pi^{00}$ can be done for a discrete Matsubara frequency $\omega_p$, and one performs the analytic continuation $i\omega_p \rightarrow p_0$ afterwards.

- Because one is after the long distance properties of the plasma, one also makes the approximation $|\vec{p}| \ll |\vec{k}|$.

(Hard Thermal Loops: Braaten, Pisarski - 1990)

- For instance, the fermionic contribution to the spatial part $\Pi^{ij}$ of the polarization tensor reads:

$$\omega \cdot p = -\frac{g^2 N_f T}{2} \int \frac{d^3 \vec{k}}{(2\pi)^3} \tilde{\nu}_k \frac{\partial n_F(\vec{k})}{\partial k^l} \left[ \delta_{jl} - \frac{\tilde{\nu}_k^i \tilde{\nu}_k^l}{\omega - \tilde{\nu}_k \cdot \vec{p} + i\epsilon} \right]$$

($\tilde{\nu}_k \equiv \vec{k}/|\vec{k}|$, $N_f =$ number of quark flavors)

- Note: with the gluon loop, the only change is $N_f \rightarrow N_f + 2N_c$.
Quasi-particles

The functions $\Pi_{T,L}(P)$ read:

$$\Pi_T(P) = \frac{e^2 T^2}{6} \left[ \frac{p_0^2}{p^2} + \frac{p_0}{2p} \left(1 - \frac{p_0^2}{p^2}\right) \ln \left(\frac{p_0 + p}{p_0 - p}\right) \right]$$

$$\Pi_L(P) = \frac{e^2 T^2}{3} \left[1 - \frac{p_0^2}{p^2}\right] \left[1 - \frac{p_0}{2p} \ln \left(\frac{p_0 + p}{p_0 - p}\right) \right]$$

Quasi-particles correspond to poles in the propagator. Their dispersion relation is the function $p_0 = \omega(\vec{p})$ that defines the location of the pole.

The inverse of the imaginary part of $p_0$ is the lifetime of the quasi-particles (If $\text{Im}(p_0) = 0$, they are stable). In order to be able to talk about quasi-particles, one must have $\text{Im}(p_0) \ll \text{Re}(p_0)$.
Quasi-particles

- Dispersion curves of particles in the plasma:

- Thermal masses due to interactions with the other particles in the plasma:

\[ m_q \sim m_g \sim gT \]

- At this order, the quasi-particles are stable
In the complex plane of $\omega/|\vec{p}|$, the dressed propagator has poles (quasi-particles) and a cut (Landau damping):
Debye screening

- A test charge polarizes the particles of the plasma in its vicinity, in order to screen its charge:

\[ V(r) = \exp\left( - \frac{m_{\text{debye}}}{m_{\text{debye}}} r \right) \]

- The Coulomb potential of the test charge decreases exponentially at large distance. The effective interaction range is:

\[ \ell \sim \frac{1}{m_{\text{debye}}} \sim \frac{1}{gT} \]

- Note: static magnetic fields are not screened by this mechanism (they are screened over length-scales \( \ell_{\text{mag}} \sim \frac{1}{g^2T} \))
Debye screening

- Place a quark of mass $M$ at rest in the plasma, at $\vec{r} = 0$

- Scatter another quark off it. The scattering amplitude reads

\[
\mathcal{M} = \left[ g\bar{u}(\vec{k}')\gamma_\mu u(\vec{k}) \right] \left[ g\bar{u}(\vec{P}')\gamma_\nu u(\vec{P}) \right] \sum_{\alpha=T,L} \frac{P_\alpha^{\mu\nu}(Q)}{Q^2 - \Pi_\alpha(Q)}
\]

- If $\vec{P} = 0$ (test charge at rest), only $\alpha = L$ contributes

- From $(P + Q)^2 = M^2$, we get a $2\pi\delta(q_0)/2M$

- For the scattering off an external potential $A^\mu$, the amplitude is

\[
\mathcal{M} = \left[ g\bar{u}(\vec{k}')\gamma_\mu u(\vec{k}) \right] A^\mu(Q)
\]

- Thus, the potential created by the test charge at rest is:

\[
A^\mu(Q) = g \frac{\bar{u}(\vec{P}')\gamma_\nu u(\vec{P})}{2M} \frac{2\pi\delta(q_0)P_L^{\mu\nu}(0, \vec{q})}{\vec{q}^2 + \Pi_L(0, \vec{q})} = \frac{2\pi g\delta^{\mu0}\delta(q_0)}{\vec{q}^2 + \Pi_L(0, \vec{q})}
\]
Debye screening

- By a Fourier transform, we obtain the Coulomb potential:

\[ A^0(t, \vec{r}) = g \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{e^{i\vec{q} \cdot \vec{r}}}{\vec{q}^2 + \Pi_L(0, \vec{q})} \]

- If we are in the vacuum, \( \Pi_L = 0 \), and the Fourier transform gives the usual Coulomb law:

\[ A^0_{\text{vac}}(t, \vec{r}) = g \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{e^{i\vec{q} \cdot \vec{r}}}{\vec{q}^2} = \frac{g}{4\pi |\vec{r}|} \]

- In a plasma, \( \Pi_L(0, \vec{q}) = \frac{g^2 T^2}{3} \equiv m_D^2 \). The Fourier transform can also be done exactly

\[ A^0(t, \vec{r}) = g \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{e^{i\vec{q} \cdot \vec{r}}}{\vec{q}^2 + m_D^2} = \frac{g}{4\pi |\vec{r}|} \ e^{-m_D |\vec{r}|} \]

\( \triangleright \) the potential is unmodified at \( r \ll 1/m_D \), but exponentially suppressed at large distance
Landau damping

- The self-energies $\Pi_{L,T}(p_0, \vec{p})$ have an imaginary part when $|p_0| \leq |\vec{p}|$. This implies that the propagation of space-like modes is attenuated.

- A wave propagating through the plasma is damped because its quanta may be absorbed by particles of the plasma:

\[ \omega_c \sim gT \]
Relativistic hydrodynamics
**Energy-momentum tensor**

- **Noether’s theorem** states that for each continuous symmetry of the Lagrangian, there is an associated conserved current $J^\mu$, such that $\partial_\mu J^\mu = 0$.

- As a consequence, the quantity

$$Q(t) \equiv \int d^3 \vec{x} \ J^0(t, \vec{x})$$

is time independent. Proof:

$$\partial_t Q(t) = \int d^3 \vec{x} \ \partial_t J^0(t, \vec{x}) = - \int d^3 \vec{x} \ \vec{\nabla}_x \cdot \vec{J}(t, \vec{x})$$

$$= - \int d^2 \vec{S} \cdot \vec{J}(t, \vec{x}) = 0$$

- Note: the spatial vector $\vec{J}$ describes the flow of the quantity $Q$ across a surface.
Energy-momentum tensor

- In a theory invariant under translations in time and position, the energy and the momentum are conserved quantities.

- For each direction $\nu$, there is a conserved current, denoted $T^{\mu \nu}$, called the energy-momentum tensor, that obeys

$$\partial_\mu T^{\mu \nu} = 0$$

- The integral over space of the zero component gives the 4-momentum of the system

$$P^\nu = \int d^3\bar{x} \ T^{0\mu}(t, \bar{x})$$

- The vector $T^{i\mu}$ ($i=1,2,3$) represents the flow of the component $\mu$ of momentum. For $\mu = 0$, this is an energy flow. For $\mu = 1, 2, 3$, this is a 3-momentum flow and it is thus related to pressure.
Consider a fluid cell at rest, of volume $\delta V$. It has an energy $\delta P^0 = \epsilon \delta V$ and a 3-momentum $\delta \vec{P} = 0$. This can be achieved if the energy momentum tensor has the following components:

$$T^{00} = \epsilon, \quad T^{0i} = 0$$

The flow of momentum $P^i$ across an element of surface $d\vec{S}$ is $dP^i = dS^j T^{ji}$. From the definition of the pressure $p$, this must be equal to $pdS^i$. Hence $T^{ij} = p\delta^{ij}$.

Therefore, in the local rest frame of the fluid:

$$T^{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$
In an arbitrary frame where the fluid 4-velocity is $v^\mu$, the energy-momentum tensor can only be built from the symmetric tensors $g^{\mu\nu}$ and $v^\mu v^\nu$. In the local rest frame ($v^\mu = (1, 0, 0, 0)$), we must recover the previous expression. Therefore:

$$T^{\mu\nu} = (p + \epsilon) v^\mu v^\nu - p g^{\mu\nu}$$

Note: this expression is valid only for an ideal fluid, with no dissipative phenomena. In a viscous fluid, there can be a transport of momentum due to the friction of fluid layers that move at different velocities. This is taken into account by additional terms in $T^{\mu\nu}$ that are proportional to the derivatives $\partial_i v^j$, multiplied by the viscosity $\eta$. 
Ideal hydrodynamics

- The fundamental equation of non viscous hydrodynamics is simply the conservation of the energy-momentum,

\[ \partial_\mu T^{\mu \nu} = 0 \]

- In the non-relativistic limit,
  - \( v^\mu \approx (1, \vec{v}) \)
  - \( \epsilon \) becomes the mass density \( \rho \)
  - the pressure \( p \) is much smaller than the energy density \( \epsilon \)

It is easy to check that the above equation is equivalent to the continuity equation for mass and to Euler’s equation:

\[
\begin{align*}
\nu = 0 : & \quad \partial_t \rho + \vec{\nabla} \cdot (\rho \vec{v}) = 0 \\
\nu = i : & \quad \partial_t (\rho v^i) + \partial_j (\rho v^i v^j) + \partial_i p = 0
\end{align*}
\]

Note : the second equation can be cast into the more familiar form

\[
\rho \left[ \partial_t + \vec{v} \cdot \vec{\nabla}_x \right] \vec{v} + \vec{\nabla}_x p = 0
\]
Ideal hydrodynamics

- In hydrodynamics, the unknown functions are:
  - $p(t, \vec{x})$, $\epsilon(t, \vec{x})$
  - $v^\mu(t, \vec{x})$ (3 unknowns only, since $v_\mu v^\mu = 1$)

- $\partial_\mu T^{\mu\nu} = 0$ gives only 4 equations

- An additional constraint comes from the equation of state of the matter under consideration, as a relation between the local pressure $p$ and energy density $\epsilon$

- An initial condition $p_0(\vec{x})$, $\epsilon_0(\vec{x})$, $\vec{v}_0(\vec{x})$ must be specified at a certain time $t_0$. Since the relativistic Euler equation contains only first derivatives in time, this is sufficient to obtain the solution at any time $t > t_0$. 
Sound propagation

Consider a small perturbation on top of a static fluid:

\[ p = p_0 + p' \]
\[ \epsilon = \epsilon_0 + \epsilon' \]

The Euler equation, linearized in the perturbations, read:

\[ \partial_t \epsilon' + (p_0 + \epsilon_0) \nabla_x \cdot \vec{v}' = 0 \]
\[ (p_0 + \epsilon_0) \partial_t \vec{v}' + \nabla_x p' = 0 \]

Differentiate the 1st equation with respect to time, and eliminate the velocity \( \vec{v}' \). We get:

\[ \partial_t^2 \epsilon' = \nabla_x^2 p' \]

For small perturbations, write \( \epsilon' = (\partial \epsilon / \partial p)_0 p' \). Therefore,

\[ \frac{1}{c_s^2} \partial_t^2 p' = \nabla_x^2 p' \quad \text{with} \quad c_s^2 \equiv \left( \frac{\partial p}{\partial \epsilon} \right)_0 \]
Phenomenology
Initial energy density

- Bjorken estimate:

\[ \epsilon_0 \approx \frac{1}{S_\perp \tau_0} \frac{dE_\perp}{dy} \]

- \( dE_\perp/dy \approx 620 \text{ GeV} \) at RHIC (\( \sqrt{s} = 200 \text{ GeV} \), gold nuclei)

- \( S_\perp \approx 140 \text{ fm}^2 \) for central collisions

- \( \tau_0 \approx 0.15 \text{ fm} \)

\[ \therefore \epsilon_0 \approx 30 \text{ GeV/fm}^3 \]

- Reminder: lattice QCD predicts deconfinement at \( \epsilon_{\text{crit}} \sim 1 \text{ GeV/fm}^3 \)

- Note: things look less impressive in terms of the temperature since \( \epsilon \sim T^4 \Rightarrow T/T_{\text{crit}} \sim 30^{1/4} \sim 2.3 \)
Thermal photons

- Photons produced by the QGP:
  - Rate determined by physics at the scale $g^2 T$
  - Very sensitive to the temperature: $dN_\gamma / dt d^3 \vec{x} \sim T^4$
Thermal photons

- Photons produced by the QGP:
  - Rate determined by physics at the scale $g^2 T$
  - Very sensitive to the temperature: $dN_\gamma/dtd^3\vec{x} \sim T^4$

- But very important background...
  - initial photons
  - photons produced by in-medium jet fragmentation
  - photons produced by the hadron gas
  - meson decays
Direct photons at RHIC

\[
d^2N/(\pi dp_T^2 dy) \text{ (GeV/c)}^{-2}
\]

**Au+Au \rightarrow \gamma+X [0-10\% central]**

- Total: Prompt + Thermal
- Prompt: NLO pQCD \times T_{AA}[0-10\%]
- Thermal: QGP+HRG
- QGP
- HRG
- PHENIX data

**Graph:**
- Data points for PHENIX data are plotted.
- Theoretical curves represent different contributions (Prompt, Thermal, QGP, HRG).
- The graph shows the distribution of direct photons as a function of transverse momentum \(p_T\) and rapidity \(y\).

**Legend:**
- Different colors and line styles represent various theoretical models or combinations of them.

**Axes:**
- \(p_T\) (GeV/c) on the x-axis.
- \(d^2N/(\pi dp_T^2 dy)\) on the y-axis.
High $p_{\perp}$ jets are produced at the initial impact

- Not very interesting by themselves...
High $p_\perp$ jets are produced at the initial impact
- Not very interesting by themselves...

**Radiative energy loss** when they travel through the QGP
- Sensitive to the energy density of the medium
- Depends on the path length as $L^2$
- Important modification of the azimuthal correlations
Hadrons are strongly suppressed
- Mesons involving heavy quarks (e.g. $D$) are also suppressed
- Photons are not suppressed

The correlation at 180° disappears in AA collisions
QGP “opacity”

- Interpretation:
  - Jets escape only if they are produced near the edge and are directed outwards.
  - The opposite jet is totally absorbed.
    ▶ confirms the very large energy density.
Collective flow

- Consider a non-central collision:
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- Initially, the momentum distribution of particles is isotropic in the transverse plane, because their production comes from local partonic interactions.
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- If these particles were escaping freely, the distribution would remain isotropic at all times.
Collective flow

Consider a non-central collision:

- Initially, the momentum distribution of particles is isotropic in the transverse plane, because their production comes from local partonic interactions.
- If these particles were escaping freely, the distribution would remain isotropic at all times.
- If the system has a small mean free path, pressure gradients are anisotropic and induce an anisotropy of the distribution.
Collective flow and ideal hydrodynamics

- Observable: 2nd harmonic of the azimuthal distribution

\[ \frac{dN}{d\varphi} \sim 1 + 2v_1 \cos(\varphi) + 2v_2 \cos(2\varphi) + \cdots \]

\( v_2 \) measures the ellipticity of the momentum distribution

- Note: even heavy quarks seem to follow this flow
Another success of hydrodynamics

- Hydrodynamics reproduces the hadron spectra at low $p_{\perp}$
Is the QGP a perfect fluid?

- Note: a **perfect fluid** is a fluid with a **very small viscosity**, that can be described with Euler equations ([ideal hydrodynamics](#)).

- The elliptic flow coefficient $v_2$ measured at RHIC is well reproduced by ideal hydrodynamics, that has no viscosity.
  - In hydrodynamics, the relevant parameter is the dimensionless ratio $\eta/s$ of the shear viscosity to the entropy density.
  - It has been concluded from there that the QGP must have a very small ratio $\eta/s$.

- In the weakly coupled QGP, $\eta/s$ is all but small...
Statistical models

- One assumes that particles are produced by a thermalized system with temperature $T$ and baryon chemical potential $\mu_B$.

- The number of particles of mass $m$ per unit volume is:

$$\frac{dN}{d^3 \vec{x}} = \int \frac{d^3 \vec{p}}{(2\pi)^3} \frac{1}{e^{(\sqrt{p^2+m^2}-\mu_B Q)/T} \pm 1}$$

- These models reproduce the ratios of particle yields with only two parameters.

- The same models also work for $e^+e^-$ collisions:
  - Standard explanation: randomly filling a phase space leads to exponential distributions.
  - However, this argument alone does not explain why the value of $T$ that comes out is the same as in nucleus-nucleus collisions. Dynamical arguments (about the properties of the vacuum?) may be involved here...
Freeze-out parameters

![Graph showing freeze-out parameters with points for RHIC, SPS, AGS, and SIS. The graph plots $T_f$ [MeV] against $\mu_B^f$ [GeV]. The line $\langle E \rangle / \langle N \rangle = 1$ GeV is also shown.]

**Legend:**
- RHIC
- SPS
- AGS
- SIS
- $\langle E \rangle / \langle N \rangle = 1$ GeV
Strangeness enhancement

- In a nucleon, the distribution of strange quarks is smaller than that of $u, d$ quarks (valence) by a factor of the order of $\alpha_s \sim 0.2–0.3$
  - In $pp$ collisions, less strange particles are produced than non-strange particles

- In the QGP, the average energy of $u, d$ quarks and of the gluons is of the order of the temperature
  - If $T$ is large enough (compared to the mass of the strange quark), then the processes $u\bar{u} \rightarrow s\bar{s}$, $d\bar{d} \rightarrow s\bar{s}$, $gg \rightarrow s\bar{s}$ are not inhibited by the kinematical threshold due to the mass of the $s$ quark

- In this case, the population of strange quarks will become identical to that of light quarks
  - The production of strange hadrons will be enhanced compared to proton-proton collisions

- The interpretation of data based on statistical models works also for strange particles at RHIC
Strangeness enhancement

![Graph showing strangeness enhancement](image)

- Collective phenomena
- Relativistic hydrodynamics
- Phenomenology
  - Initial energy density
  - Initial temperature
  - QGP "opacity"
  - Collective flow
  - Hadronization
- Strangeness
- Deconfinement

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Strangeness enhancement

STAR Preliminary

Yield/$N_{\text{part}}$ relative to pp/Be

1

10

100

$N_{\text{Part}}$

$\Omega + \Omega$

$\Lambda$

$\Xi^-$

$\Xi^+$

$N_{\text{part}}$

$\Lambda$

$\Xi^-$

NA57

STAR

Initial energy density
Initial temperature
QGP "opacity"
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Lecture II / II – Master 2ème année - spécialité NPAC, Orsay, France, March 2009 - p. 41/45
Debye screening prevents the $Q\bar{Q}$ pair from forming a bound state \cite{MatsuiSatz1986}.
- each heavy quark pairs with a light quark in order to form a $D$ meson.

The inter-quark potential can be calculated using lattice QCD.

Possible observable: $[J/\psi] / [\text{Open charm}]$
- complication: there is also a suppression in proton-nucleus collisions, due to multiple scattering.
J/Psi suppression

- What do we do with this potential?
  - Shröedinger equation for a \( Q\bar{Q} \) bound state:
    \[
    \left[ 2m_Q + \frac{1}{m_Q} \nabla^2 + U(r, T) \right] \Psi = M(T)\Psi
    \]
  - Non-relativistic
  - Assumes that there are only two-body interactions

- Dissociation temperatures:
  - Dissociation temperatures:
    - \( T_d/T_c \):
      - \( J/\psi \): 2.0
      - \( \chi_c \): 1.1
      - \( \psi' \): 1.1
      - \( \Upsilon \): 4.5
      - \( \chi_b \): 2.0
      - \( \Upsilon' \): 2.0
  - the \( Q\bar{Q} \) states are not dissolved immediately above the critical temperature
... or enhancement?

- Many $Q\overline{Q}$ pairs may be produced in each $AA$ collision
  - Braun-Munzinger, Stachel (2000)
  - Thews, Schroedter, Rafelski (2001)
    - A $Q$ from one pair may recombine with a $\overline{Q}$ from another pair
- Avoids the conclusion of Matsui and Satz’s scenario, provided that the average distance between heavy quarks is smaller than the Debye screening length
- May lead to an enhancement of $J/\psi$ production
J/Psi measurements at RHIC

Collective phenomena
Relativistic hydrodynamics
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- Initial energy density
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