Quantum dimer models on the kagome lattice

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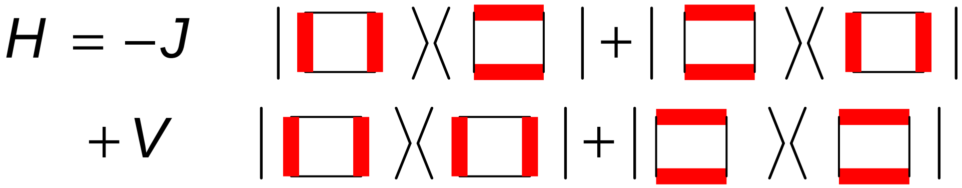
Phys. Rev. Lett **89**, 137202 (2002) [cond-mat/0204428]

Phys. Rev. B **67**, 214413 (2003) [cond-mat/0302152]

J. Phys. Cond. Mat. **16**, 823 (2004) [cond-mat/0310661]

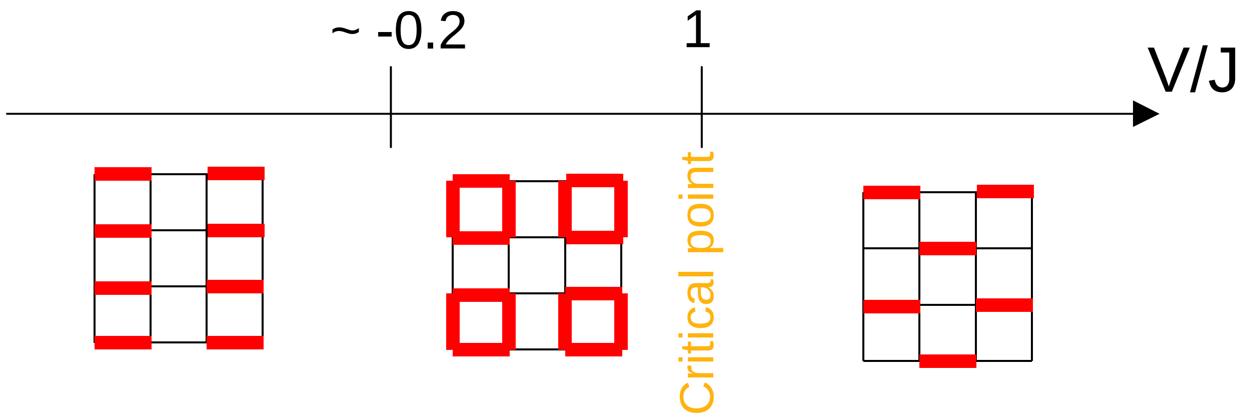
Square lattice QDM

Rokhsar & Kivelson PRL '88



Columnar valence-bond crystal (VBC)

Staggered crystal

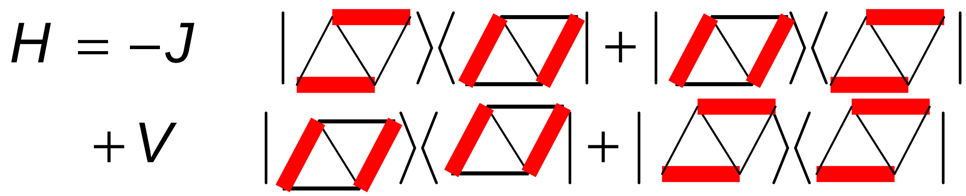


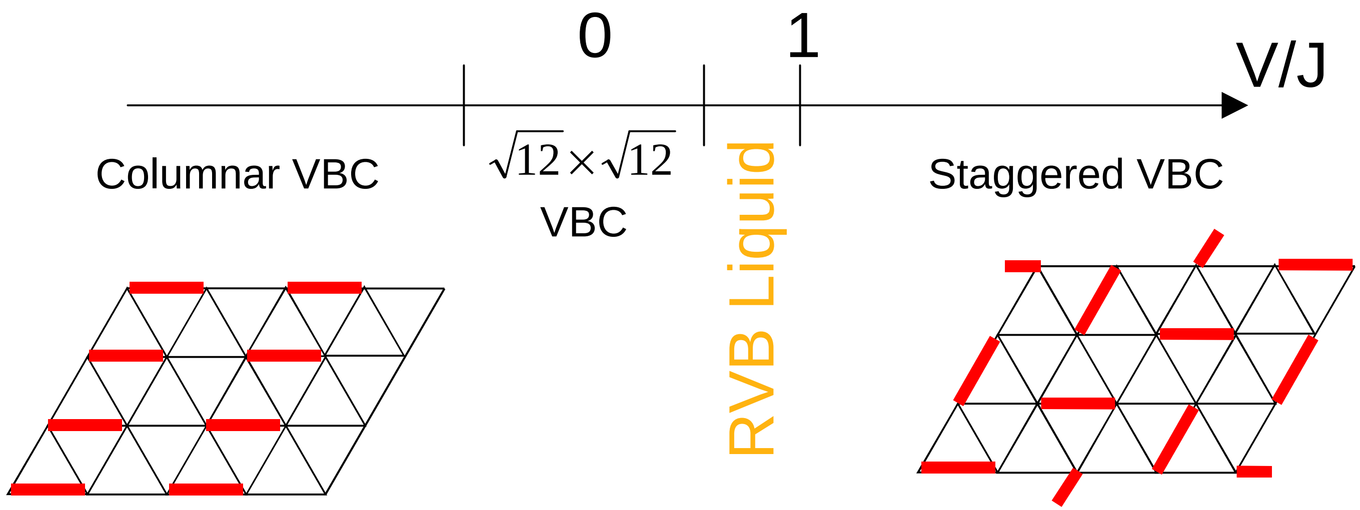
L. S. Levitov, Phys. Rev. Lett. ***64****, 92 (1990)*
S. Sachdev, Phys. Rev. B ***40****, 5204 (1989)*
P. W. Leung, et al., Phys. Rev. B ***54****, 12 938 (1996)*

No genuine liquid RVB phase...

Triangular lattice QDM

Moessner & Sondhi PRL '01

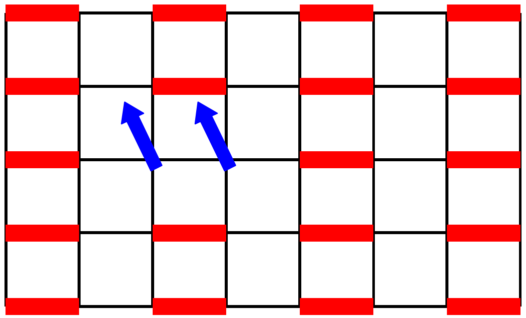


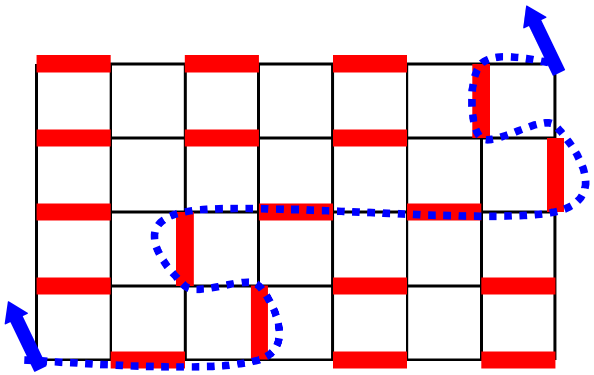


But what is so remarkable with RVB liquids ?

Spinon deconfinement in RVB liquids

- Crystal

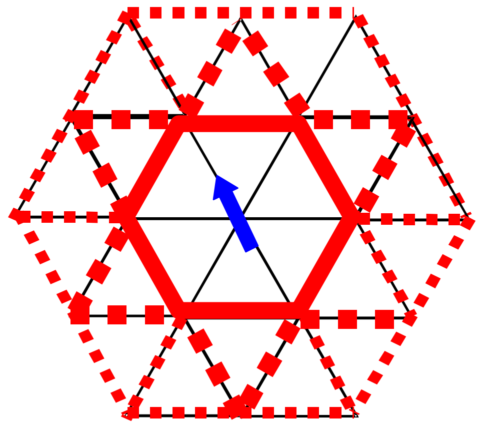




Energy grows linearly with distance
confinement

(different from 1D)

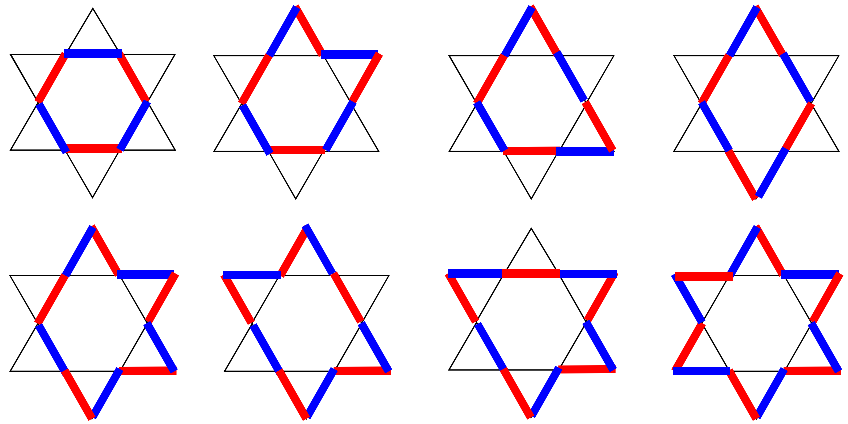
- Liquid
  - No broken symmetry
  - Short-ranged correlations



One spinon is surrounded by a local reorganization of the (liquid-like) dimer background.
Deconfinement

This work: a *solvable* model with a RVB liquid ground-state

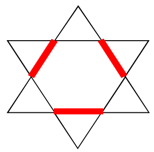
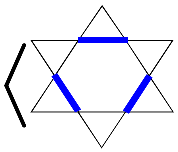
Single hexagon moves on the kagome lattice



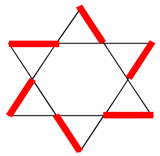
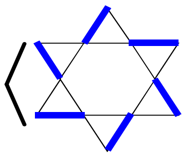
32 different loops when applying the hexagon symmetries

Any other move will involve at least 2 hexagons

Kinetic energy operator σ^x : linear combination of all these possible moves:

$\sigma^x = |$  $\rangle\langle$  $| + \text{h.c.}$

$+$

$+ |$  $\rangle\langle$  $| + \text{h.c.}$

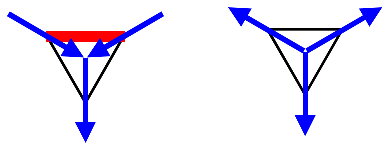
Solvable QDM

$$H = -\sum_{h \in \text{hexagons}} \sigma^x(h)$$

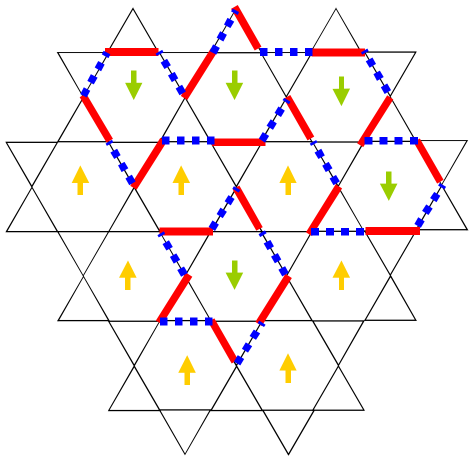
Spectrum and wave-functions ?

Need for two representations:

- Arrows



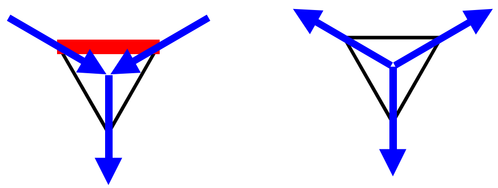
- Pseudo-spins

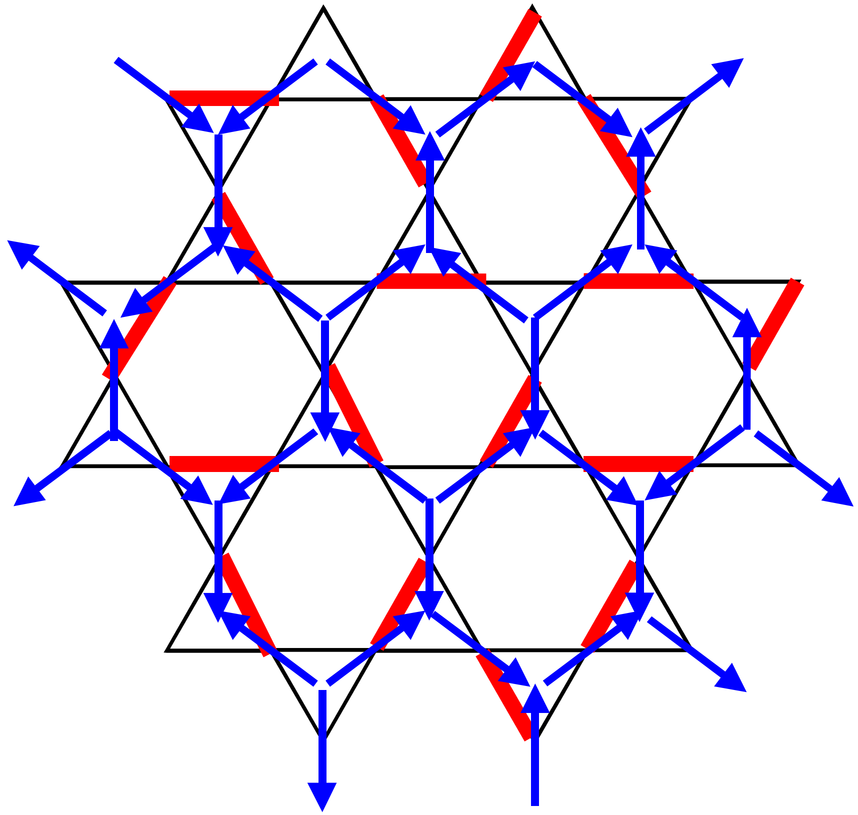


Arrow representation

Zeng & Elser PRB '93 '95

On a lattice made of corner-sharing triangles (such as kagome), dimer coverings are easily represented with *arrows*:





Constraint :

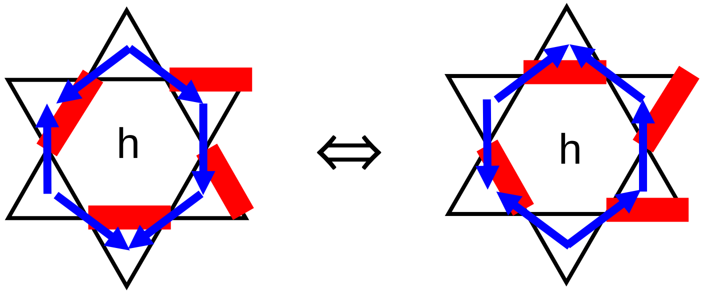
- Number of outgoing arrows must be *odd* on every triangle
- Flipping the arrows along any closed loop is an admissible move.

Relation to our model ?

Arrows and σ^{x}

In terms of the arrows, σ^{x} has a very simple meaning:

$\sigma^x(h)$: Flips the 6 arrows around *h*



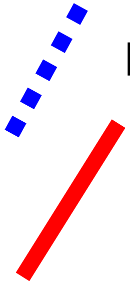
ð $\sigma^x(h)^2 = 1$ $\qquad [\sigma^x(h), \sigma^x(h')] = 0 \quad \forall h, h'$

σ^{x} ⇔ Ising pseudo-spin

σ^z pseudo-spin operators

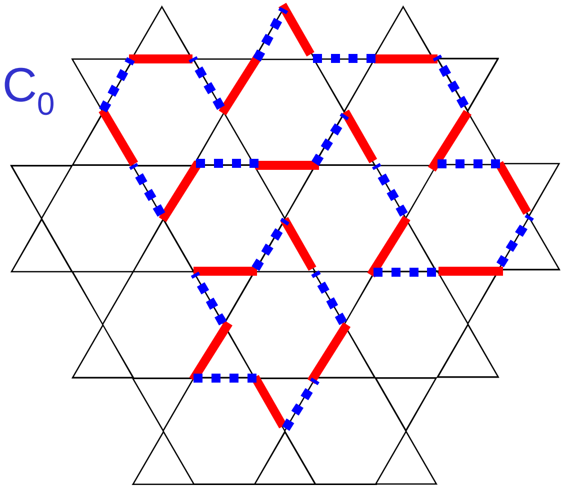
Zeng & Elser PRB '93 '95

Label dimer coverings by $\sigma^z = \pm 1$ on each hexagon :

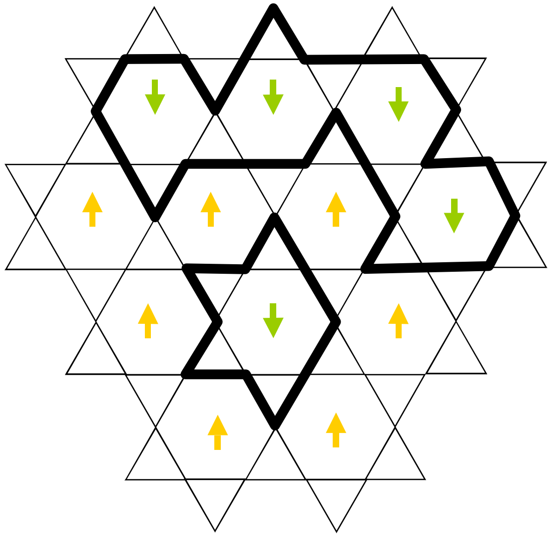


Reference covering C_0

New covering C



Related to the solution of the 2D Ising model with dimers on a decorated lattice.
M. E. Fisher, J. Math. Phys. ***7****, 1776 (1966)*



= non-intersecting loops
= Ising domain walls $\sigma^z(h)=\pm 1$

Remarks in case of periodic boundary conditions:

- C and C_0 must be in the same topological sector (all loops are contractible).

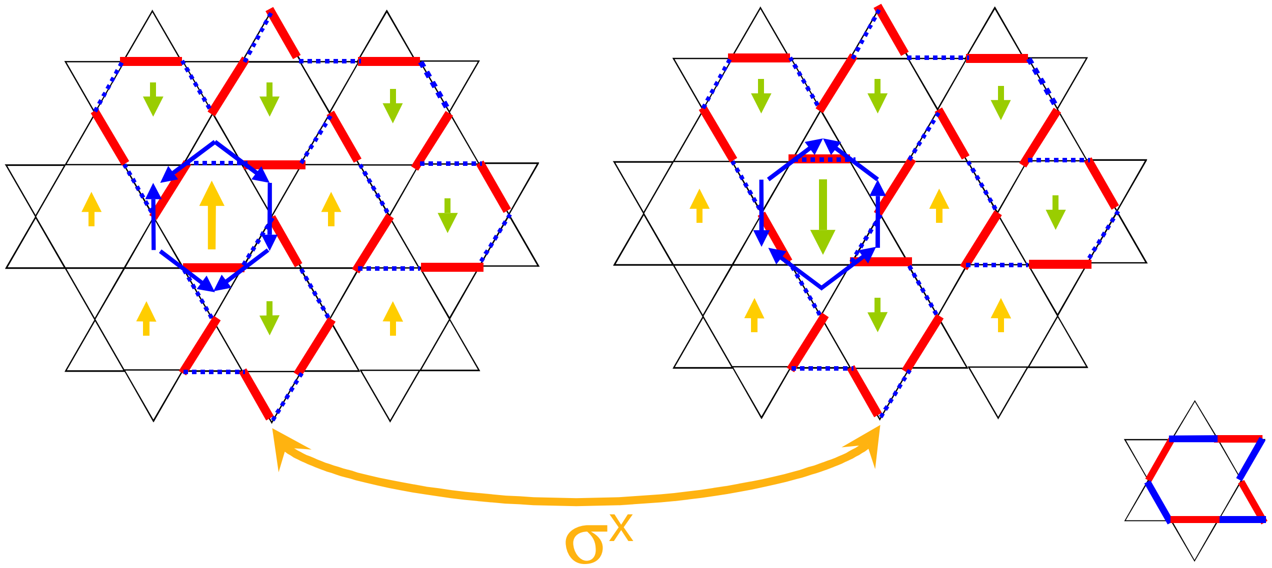
- $\sigma^z(h)$ is defined modulo a global spin flip

σ^x and σ^z

The pseudo-spin operators satisfy the usual Pauli matrix algebra:

$$\sigma^x(h)\sigma^z(h) = -\sigma^z(h)\sigma^x(h)$$

$$\sigma^x(h),\sigma^z(h') \quad = 0 \qquad \forall h \neq h'$$



Ground-state wave function

$$H = - \underset{h \in \text{hexagons}}{\quad} \sigma^x(h)$$

$$|0\rangle = |\rightarrow \quad \rightarrow \qquad \rightarrow\rangle$$

$$= \big(|\uparrow\rangle + |\downarrow\rangle\big) \; \big(|\uparrow\rangle + |\downarrow\rangle\big) \qquad \big(|\uparrow\rangle + |\downarrow\rangle\big)$$

$$= \underset{c}{\quad} |c\rangle$$

= linear combination of all dimer coverings

(belonging to a given topological sector)

= Rokhsar-Kivelson wave-function

Dimer-dimer correlations :

$$\langle 0 | d_{ij} d_{kl} | 0 \rangle = (1/4)^2 = \langle 0 | d_{ij} | 0 \rangle^2 = \langle 0 | d_{kl} | 0 \rangle^2$$

(Simple proof using $\sigma^x(h)|0\rangle = |0\rangle$)

Strictly no correlation as soon as bonds (*ij*) and (*kl*) do not involve a common triangle. *Most disordered dimer liquid*.

Excitations ?

Excitations: Ising vortices (*visons*)

*N. Read and B. Chakraborty, Phys. Rev. B **40**, 7133 (1989)*
*S. Kivelson, Phys. Rev. B **39**, 259 (1989)*
*T. Senthil and M. P. A. Fisher, Phys. Rev. Lett. **86**, 292 (2001)*

$$H = - \quad \sigma^x(h)$$
$h \in$ hexagons

$\prod_h \sigma^x(h)$: Flips all the arrows twice no effect

ð Contraint on physical states: $\prod_h \sigma^x(h) = 1$

Systems without ‘edges’ (periodic boundary conditions). Related to the two-fold redundancy $\sigma^z \Leftrightarrow -\sigma^z$

ð Excitations are created by pairs

$$|ab\rangle = |\ \rightarrow \ \rightarrow \ \boxed{\leftarrow_a} \ \rightarrow \qquad \boxed{\leftarrow_b} \ \rightarrow \rangle$$

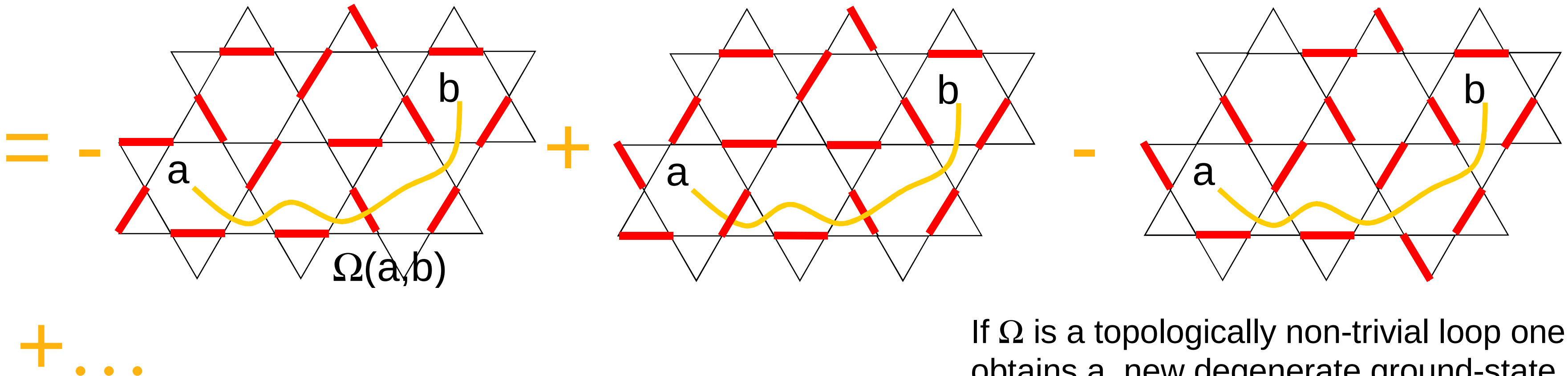
$$E_{ab} = E_0 + 4$$

Wave-function in terms of dimers ?

Ising vortex (vison) wave-function

$$H = - \quad \sigma^x(h)$$
$$h \in \text{hexagons}$$

$$|ab\rangle = |\ \rightarrow\ \rightarrow\ \boxed{\leftarrow_a}\ \rightarrow\ \quad \boxed{\leftarrow_b}\ \rightarrow\rangle$$
$$= \sigma^z(a)\sigma^z(b)|0\rangle$$
$$= \quad (-1)^{\Omega(a,b)}|c\rangle$$
$$c$$



If Ω is a topologically non-trivial loop one obtains a new degenerate ground-state (topological degeneracy)

Why are these excitations called *vortices* ?

Z_2 Gauge theory

2+1 dimensions - Hamiltonian formulation

- Gauge variable = arrows

(living on the links on the hexagonal lattice)

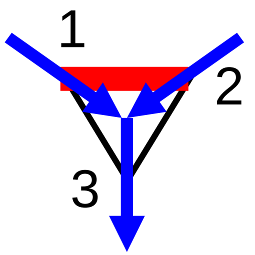
$$\tau^x(i) = \begin{cases} +1 & \text{If the arrow } \boldsymbol{i} \text{ is the same as in the reference} \\ -1 & \text{Otherwise} \end{cases}$$

$\tau^z(i) =$ Flips the arrow ***i***

- Gauge constraint

  Number of outgoing arrows must be *odd* on every triangle

$$\prod_{i=1}^{3} \tau^x(i) = 1$$



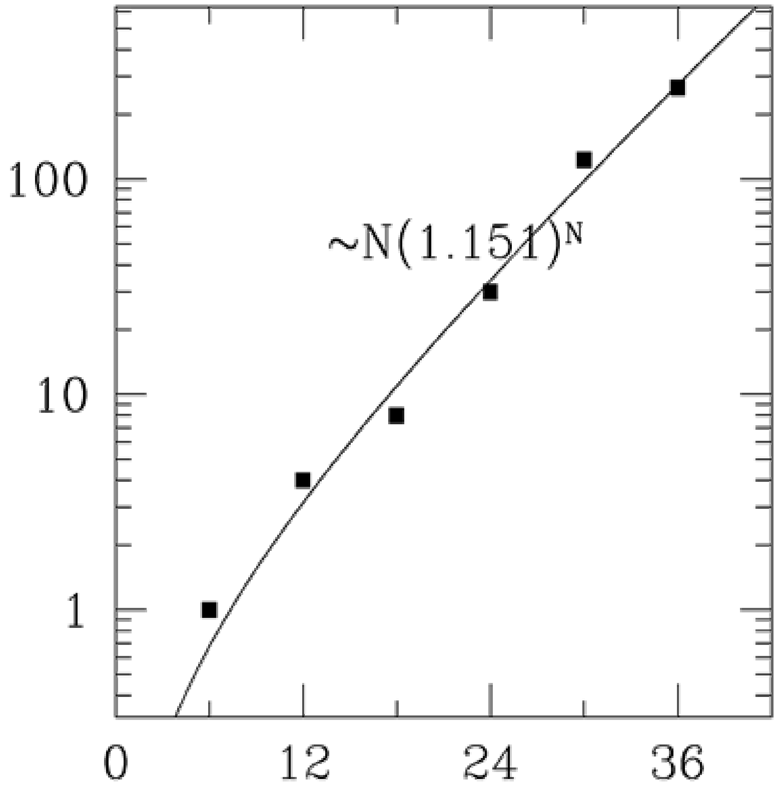
⇒ Dimer coverings are the physical states of the gauge theory.
Visons are the vortices of this gauge theory.

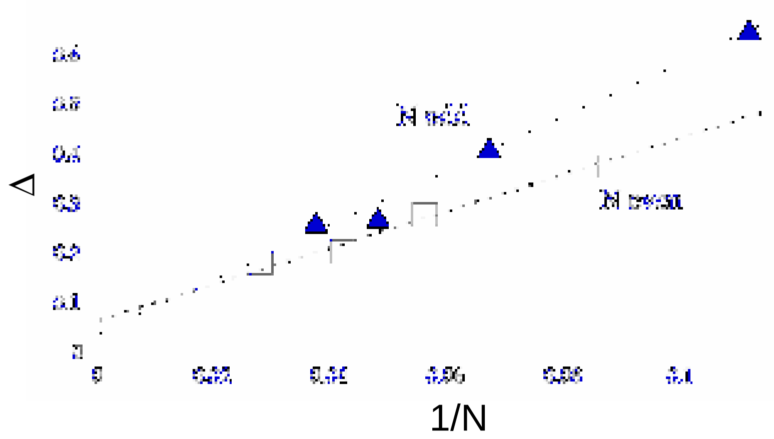
S=½ Heisenberg model on the kagome lattice

Huge number of low-energy singlets in the spectrum

Small (or vanishing ?) spin gap

No clear signature of any LRO so far



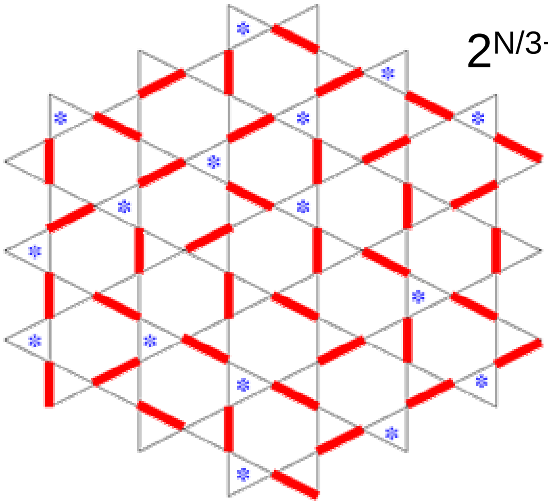


Explanation ...?

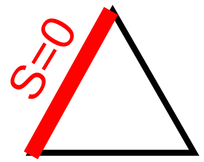
NB: Log(1.15)/log(2)=0.2

Waldtmann et al. Eur. Phys. J. B (1998)

Variational subspace of dimer coverings



$2^{N/3+1}$ coverings



= ground-state of

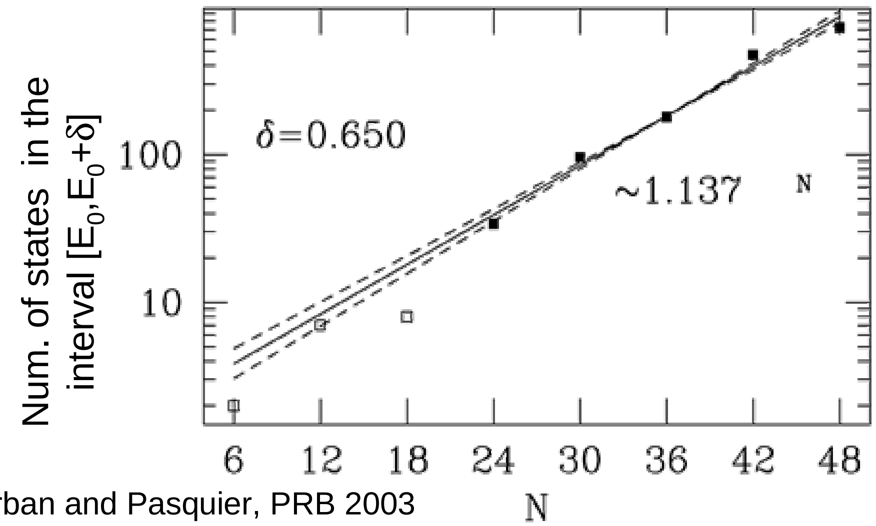
$$S_1 \cdot S_2 + S_2 \cdot S_3 + S_3 \cdot S_1$$

In a dimer covering, ¾ of the triangles have their minimal energy

Good variational starting point

Numerical diagonalization of the Heisenberg model in the dimer subspace

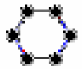
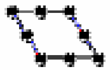
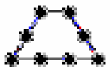
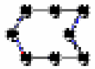
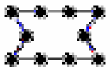
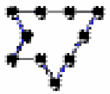
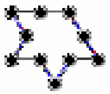
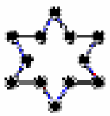
Large density of singlet states (similar to spectra in the full space)



GM, Serban and Pasquier, PRB 2003

Simplified model to describe the dimer dynamics induced by the Heisenberg interaction ?

Effective models in the dimer subspace

Loop	Heisenberg	σ^x	μ	$\tilde{\mu}$
	$-\frac{3}{4}$	1	-1	-1
	$\frac{1}{4}$	1	1	-1
	$\frac{1}{4}$	1	1	1
	$\frac{1}{4}$	1	1	-1
	$-\frac{1}{16}$	1	-1	-1
	$-\frac{1}{16}$	1	-1	1
	$-\frac{1}{16}$	1	-1	-1
	0	1	1	-1
8 simplest dimer moves (single star)	Zeng and Elser 1993	Solvable model	⇧	

$$_{\mu} = - {}_{h} \ \mu(h)$$

Sign of the matrix element = $(-1)^{\text{Num. dimers}}$

μ-algebra and extensive degeneracy

$$ {}_{\mu} = - {}_{h} \mu(h) $$

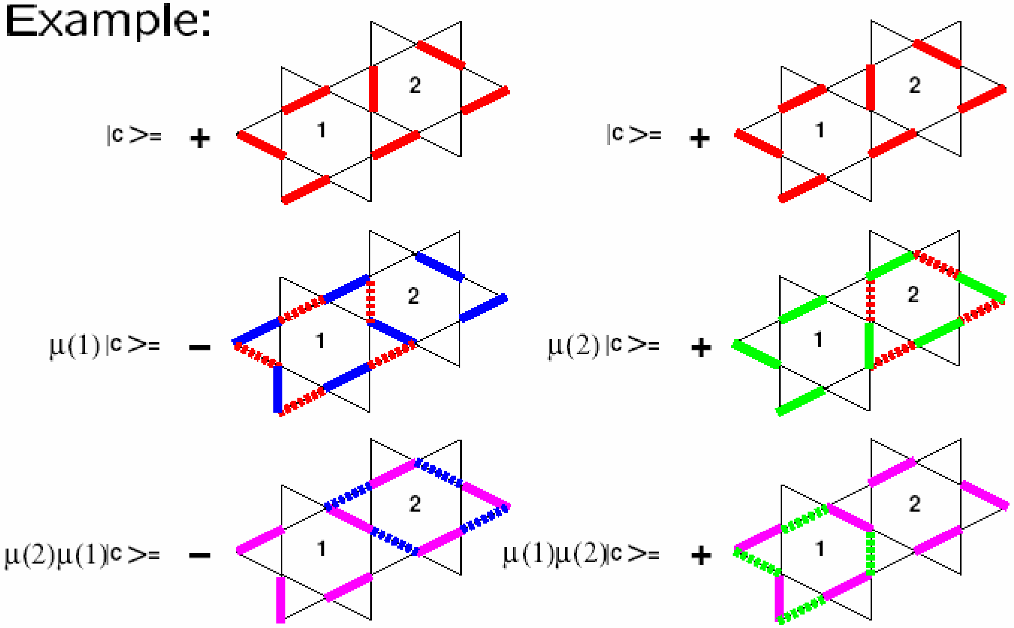
$\mu_a{}^2 = 1$ Kind of Ising pseudo-spin but...

$$\begin{cases} \mu_1\mu_2 = \mu_2\mu_1 & \text{If 1 and 2 are not neighboring hexagons} \\ \mu_1\mu_2 = -\mu_2\mu_1 & \text{If 1 and 2 \textit{are} neighboring hexagons} \end{cases}$$

Frustration

+ same for the $\tilde{\mu}$

Example:



Extensive degeneracy

$\mu = - \quad_h \quad \mu(h)$

The μ and $\tilde{\mu}$ commute with each other :

$$\forall a,b: \quad [\mu_a, \tilde{\mu}_b] = 0$$

Starting from one ground-state, one generate some others by acting with the $\tilde{\mu}$
= generators of a large non-Abelian **symmetry group**

With the $\tilde{\mu}$ one can build $N_h/2$ operators which commute with each other (as well as with H_μ).

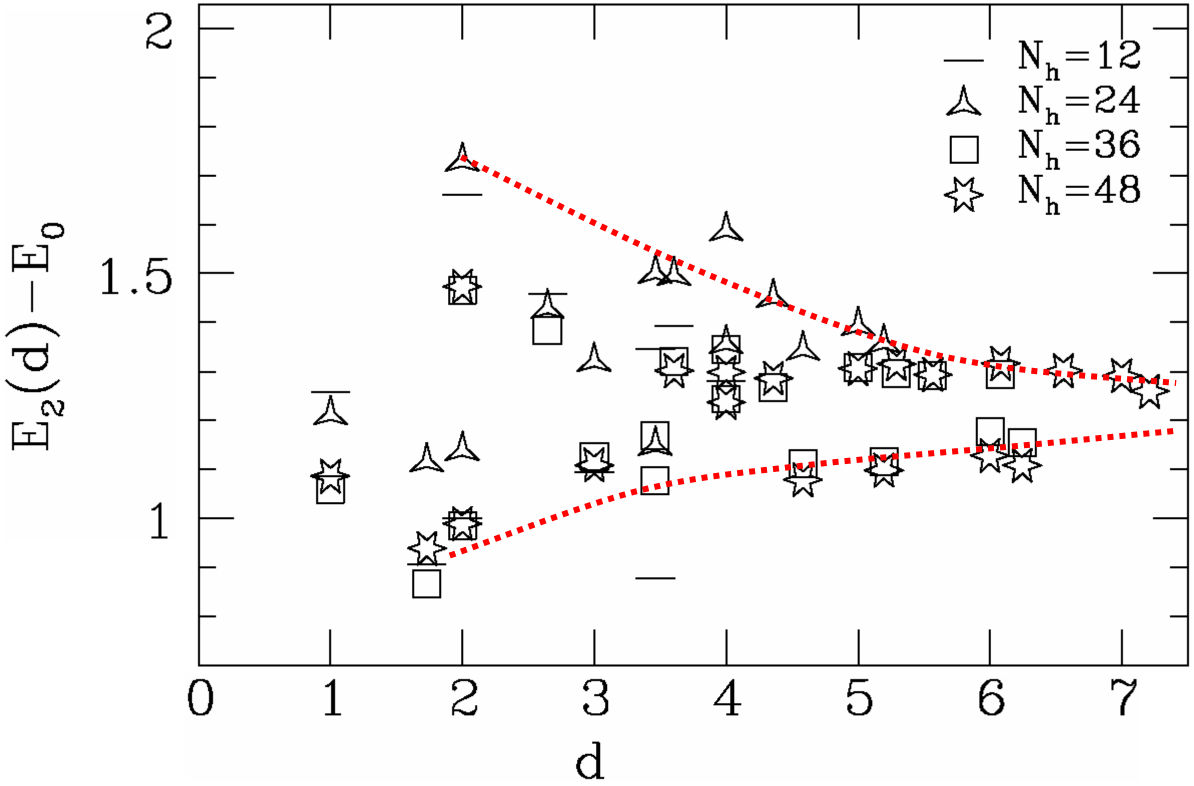
Degeneracy ~ $2^{N_h/2}$ $(=2^{N/6})$

Residual entropy (1/6)log(2) per site at T=0.

Explanation for the large density of S=0 states in the s=½ model ?

Deconfinement

Ground-state energy in presence of 2 static holes at distance d



GM, Serban and Pasquier,
J. Phys.: Cond. Mat. **16**, 823 (2004)
cond-mat/0310661

E_2(d) goes to a constant at large distance Deconfinement

See also: S. Dommange *et al.*, cond-mat/0306299 (Heisenberg S=½)
Lauechli and Poilblanc, cond-mat/0310597 (t-J model)

μ-model

$$H_\mu = - \underset{\text{hexagons}}{\quad} \mu(\mathrm{h})$$

$$\mu(h) = (-1)^{\text{Num. of dimers on } h} \, \sigma^x(h)$$

- Exact results :

1) H_μ has a (hidden) local non-Abelian symmetry.

Extensive ground-state degeneracy $\approx 2^{N/6}$!

2) Short-ranged dimer-dimer correlations Dimer liquid

- Numerics (exact diagonalizations up to 144 sites) :

1) Gapless spectrum

2) Finite-size scaling of correlations and susceptibilities suggest that the system is critical

Is there a relation between this quantum dimer model and the low-energy singlet sector of the kagome-lattice spin-½ Heisenberg model ?

Summary

- σ^x model

  - First exactly solvable quantum dimer model

  - Deconfined RVB liquid ground-state with topological degeneracy

  - Exact mapping to an Ising gauge theory

- μ model

  - Extensive ground-state degeneracy

  - Numerics   Probably critical

  - Relation with the spin-½ Heisenberg model on the kagome lattice ?