Spontaneous symmetry breaking and finite-size spectra of quantum frustrated antiferromagnets

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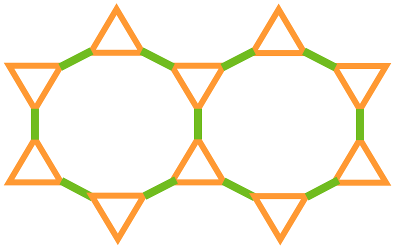
M. Oshikawa, V. Pasquier, K. Penc, L. Pierre & N. Shannon

Outline

- Exact diagonalizations and quantum numbers
- Spontaneous symmetry breaking
- Discrete SSB: some examples of valence-bond crystals

  Kagome lattice, expanded kagome lattice, others…

- Continuous SSB: Néel and nematic phases



Exact diagonalizations and quantum numbers

- Quantum lattice model ⇒ symmetry group G
- G ⇒ Irreducible representations (irreps)

  Possible “automatic” implementation. Using GAP/GRAPE for example (`www.gap-system.org`)

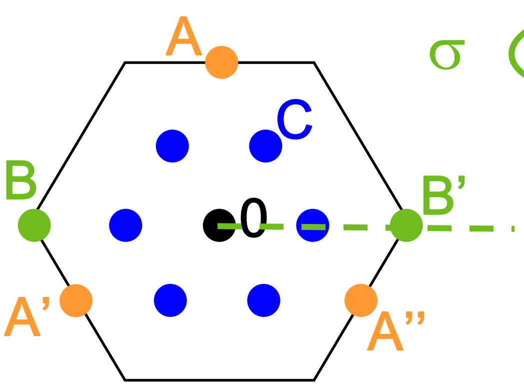
- Irrep. ⇒ block diagonalization

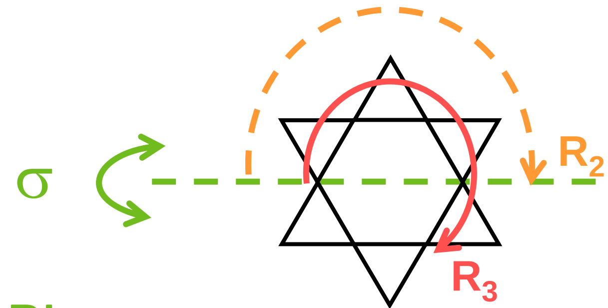
  In most practical cases, irreps are induced by *1d representations* of some subgroup.

  First fix momentum **k**, then look for irrep. of the “little” (point) group of **k**

- Example. Irreps of the first levels of a 36-site kagome Heisenberg model

Number	$2\langle \vec{S}_i \cdot \vec{S}_j \rangle$	k	R_3	R_2	σ	Deg.
1	-.43837653	0	1	1	1	1
2	-.43809562	B	$e^{\pm 2i\pi/3}$			4
3	-.43807091	0	$e^{\pm 2i\pi/3}$	1		2
4	-.43799346	0	1	1	1	1
5	-.43785105	C			1	6
6	-.43758510	0	1	-1	1	1
7	-.43758455	A		1	1	3
8	-.43751941	C			-1	6
9	-.43721566	0	1	1	-1	1
10	-.43718796	0	$e^{\pm 2i\pi/3}$	1		2
11	-.43714765	A		-1	-1	3
12	-.43705108	0	$e^{\pm 2i\pi/3}$	-1		2
13	-.43703981	B	1		1	2
14	-.43703469	A		-1	1	3
15	-.43685867	0	1	-1	-1	1
16	-.43685319	B	1		-1	2
17	-.43683757	A		1	-1	3

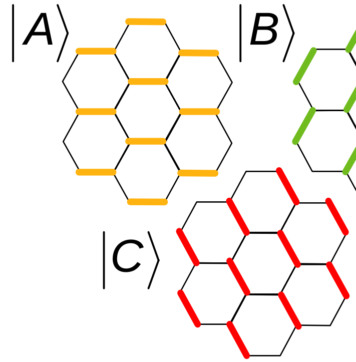


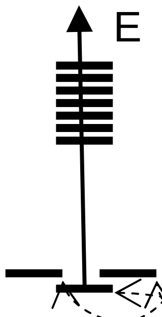


[Waldtmann, EPJB 1998 & Sindzingre, unpublished]

Spontaneous symmetry breaking and quantum numbers

- Broken symmetry states $\neq$ eigenstates





$$|2\rangle \sim |A\rangle + e^{-\frac{2i\pi}{3}}|B\rangle + e^{-\frac{4i\pi}{3}}|C\rangle$$

$$|1\rangle \sim |A\rangle + e^{\frac{2i\pi}{3}}|B\rangle + e^{\frac{4i\pi}{3}}|C\rangle$$

$$|0\rangle \sim |A\rangle + |B\rangle + |C\rangle$$

- SSB $\Rightarrow$ quasi-degenerate ground-state in finite-size system, *several* irreps.

- Group theory $\Rightarrow$ Predict the irreps associated to a given SSB phase

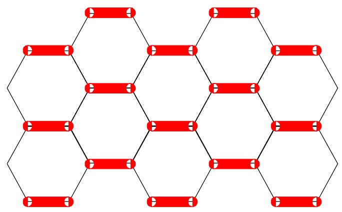
$$n_\gamma = \frac{1}{|G|}\sum_{g\in G} \underbrace{\chi_\gamma(g^{-1})}_{\text{character}} \sum_{i=1}^{d} \langle i|\hat{g}|i\rangle$$

$|A\rangle, \cdots$: Basis of the ground-state subspace

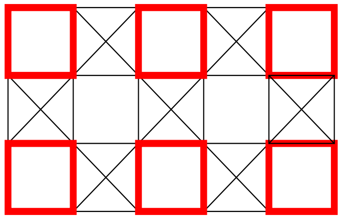
  - A simple (no fluctuations) trial ground-state can be used to compute n_γ

  - Simple for *discrete* SSB, slightly more involved for *continuous* ones

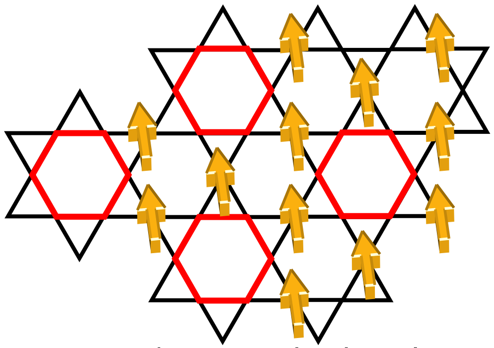
Examples of valence-bond crystals (from ED studies)



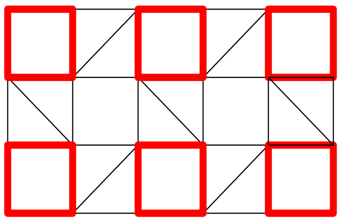
J_1-J_2-J_3 model
Fouet *et al*. EPJB 2001



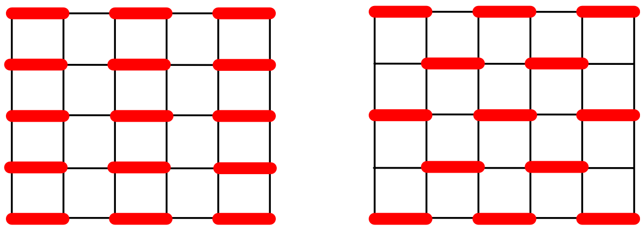
Fouet *et al*. PRB 2003



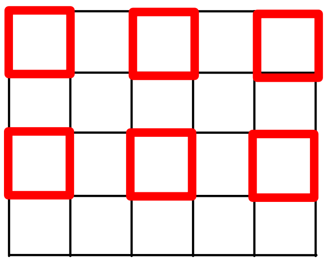
Kagome, 1/3 magnetization plateau
Cabra *et al.*, PRB 2005



Shastry-Sutherland lattice
Läuchli, Wessel & Sigrist PRB 2002



Heisenberg model & 4-spin “ring” exchange
Läuchli *et al*. PRL 2005

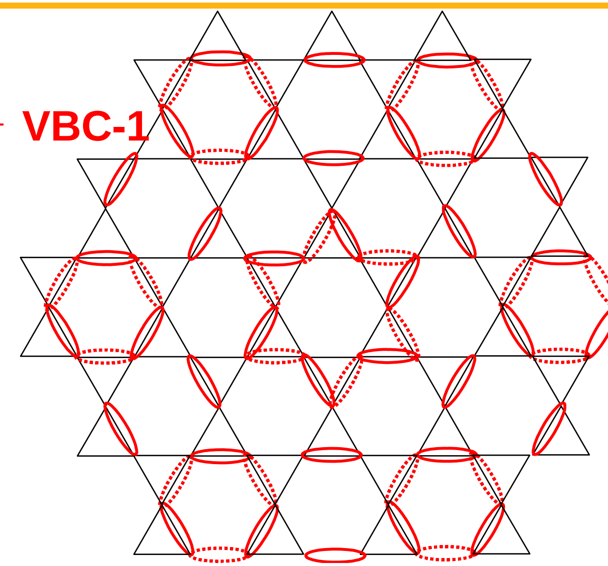


J_1-J_2-J_3 model
Mambrini *et al.*, cond-mat/0606776

+ others…

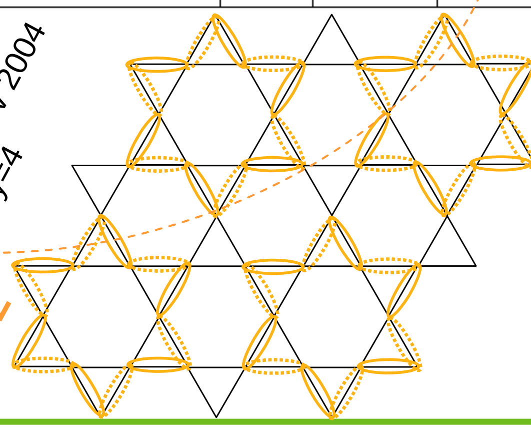
Valence bond crystals on kagome ?

	Number	$2\langle\vec{S}_i\cdot\vec{S}_j\rangle$	k	R_3	R_2	σ	Deg.	VBC-1	VBC-2	VBC-3
	1	-.43837653	0	1	1	1	1		*	*
? ⇒	2	-.43809562	B	$e^{\pm 2i\pi/3}$			4			
	3	-.43807091	0	$e^{\pm 2i\pi/3}$	1		2			*
? ⇒	4	-.43799346	0	1	1	1	1			
? ⇒	5	-.43785105	C			1	6			
	6	-.43758510	0	1	-1	1	1			*
	7	-.43758455	A		1	1	3	*		*
	8	-.43751941	C			-1	6	*		
	9	-.43721566	0	1	1	-1	1	*		*
	10	-.43718796	0	$e^{\pm 2i\pi/3}$	1		2			*
	11	-.43714765	A		-1	-1	3			*
	12	-.43705108	0	$e^{\pm 2i\pi/3}$	-1		2			*
	13	-.43703981	B	1		1	2	*		
	14	-.43703469	A		-1	1	3			*
	15	-.43685867	0	1	-1	-1	1			*
	16	-.43685319	B	1		-1	2			
	17	-.43683757	A		1	-1	3		*	*
	…	…	…	…	…	…	…			
	44	-0.43474519	0	$e^{\pm 2i\pi/3}$	-1		2			*

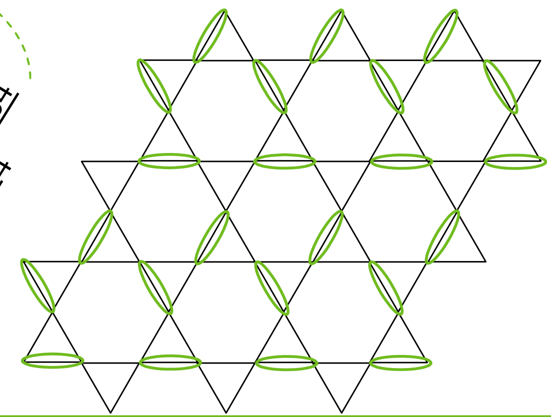


Marston & Zeng 1991; Nikolic & Senthil 2003
Degeneracy=12

Syromyatnikov & Maleyev 2004
Degeneracy=4
VBC-2

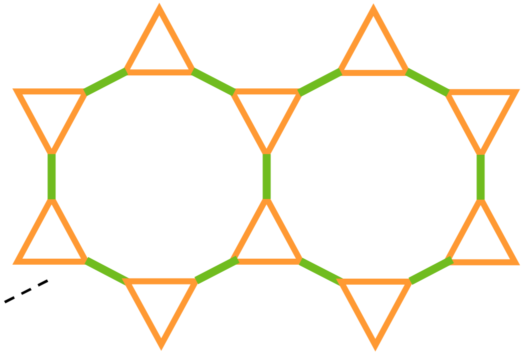


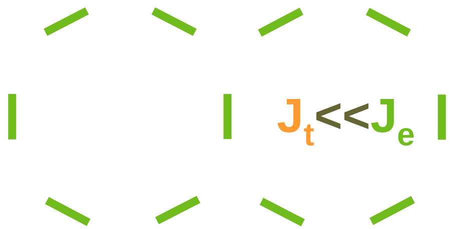
Budnik & Auerbach 2004
Degeneracy=24
VBC-3



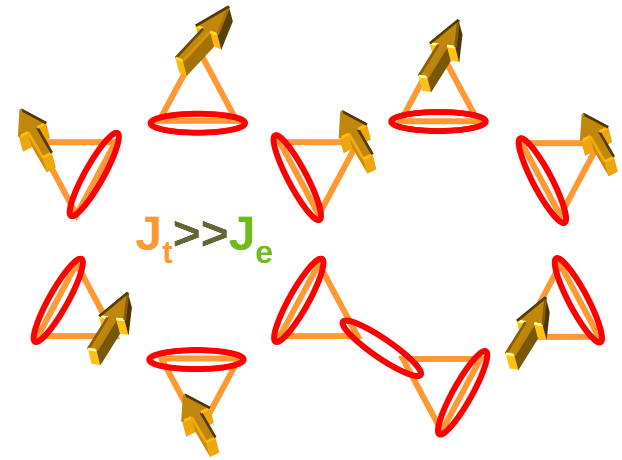
Expanded kagome (star) lattice J_e-J_t model

Similarities with kagome:
- same classical AF degeneracy
- same # of dimer coverings



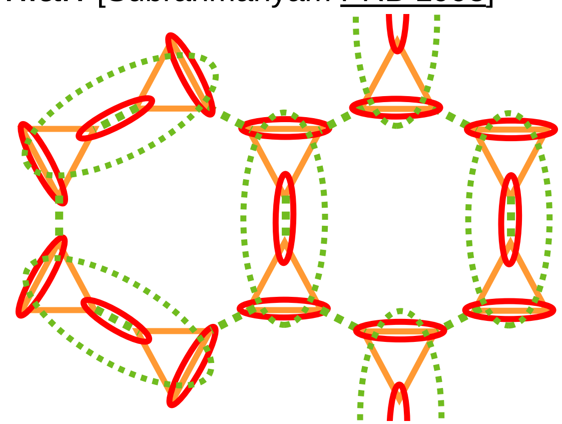


Isolated dimers - trivial limit
Single singlet ground-state
Large gap to all excitations
"quantum paramagnet"
At $J_t=J_e$, the model is in this phase
[Richter et al. PRB 2004]



Extensive degeneracy
Degenerate perturbation theory
Effective spin-chirality Hamiltonian [Subrahmanyam PRB 1995]

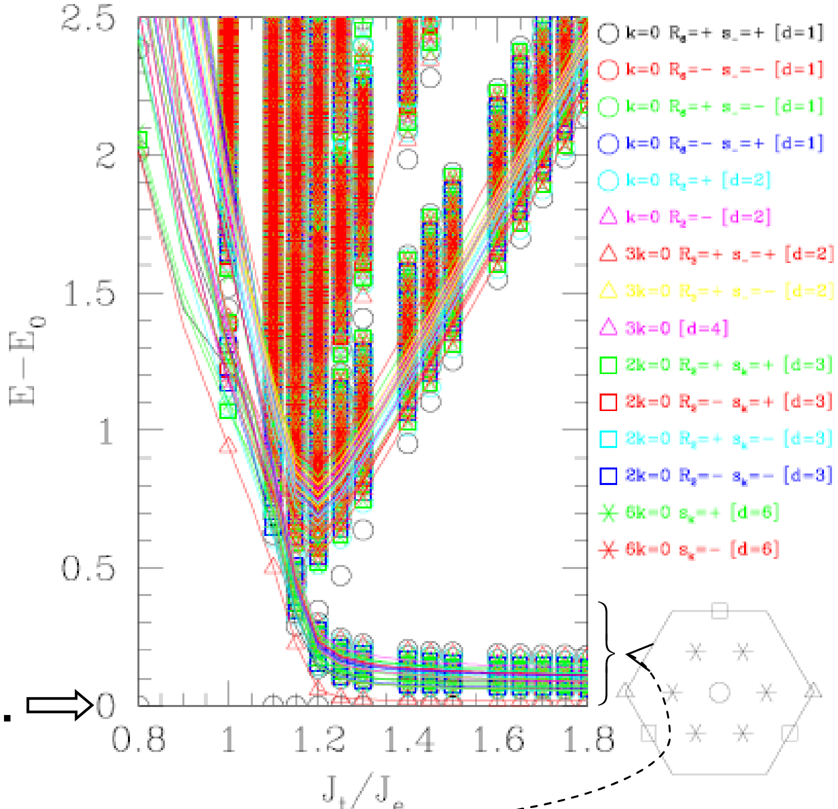
Mean-field approximation
Degenerate solutions = super coverings
[same as in Mila PRL 1998 !]



How is this degeneracy lifted beyond mean-field ?
$\Rightarrow$ We tried ED in the 1st neighbor valence-bond subspace

Expanded kagome lattice J_e-J_t model: a VBC when $J_t \gg J_e$?

ED in the 1st neighbor valence-bond subspace
[Zeng & Elser PRB 1995; Mambrini & Mila EPJB 2000]



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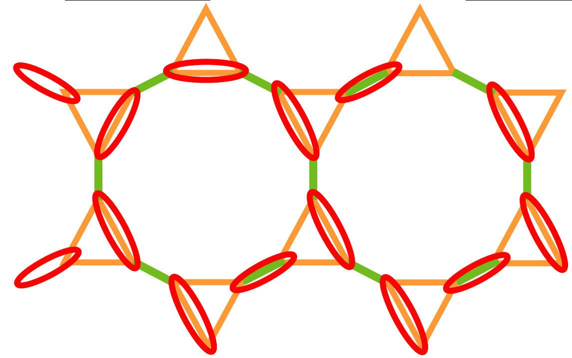
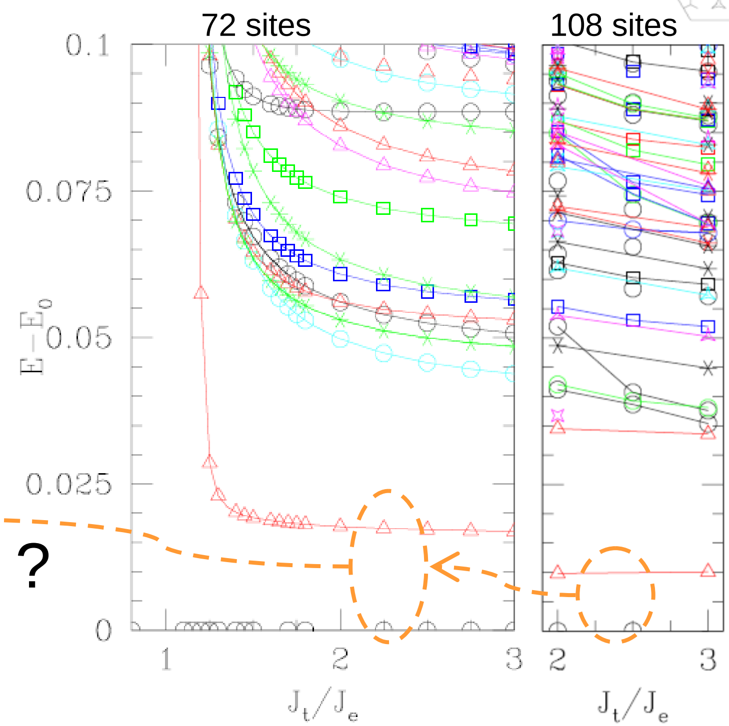
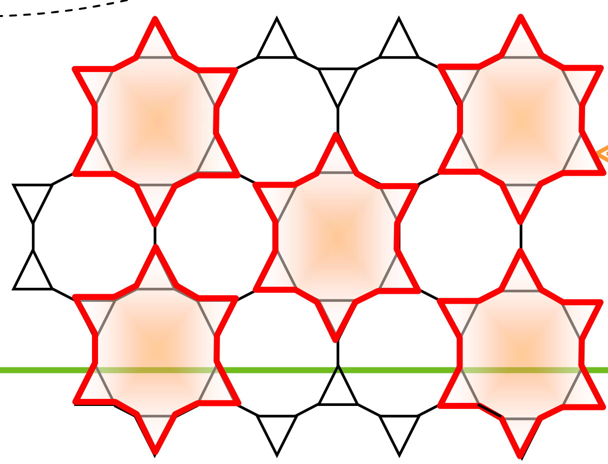


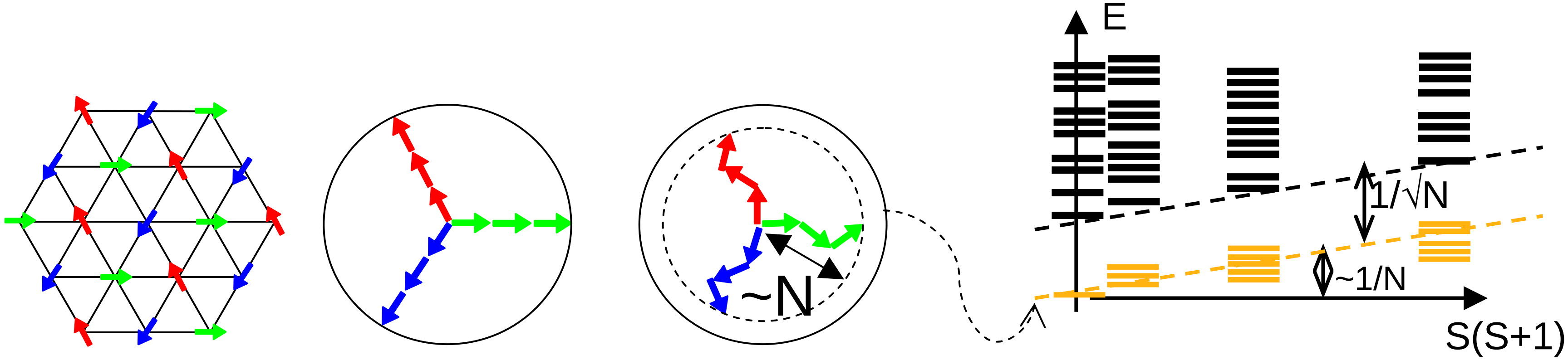




?

# of super coverings (mean-field) states

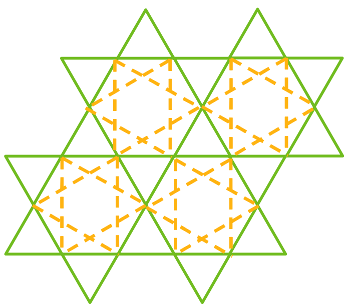
Néel phases – Anderson tower

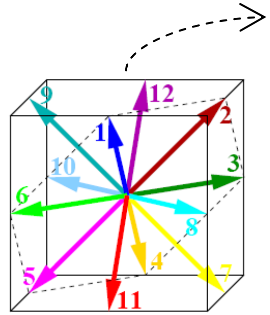


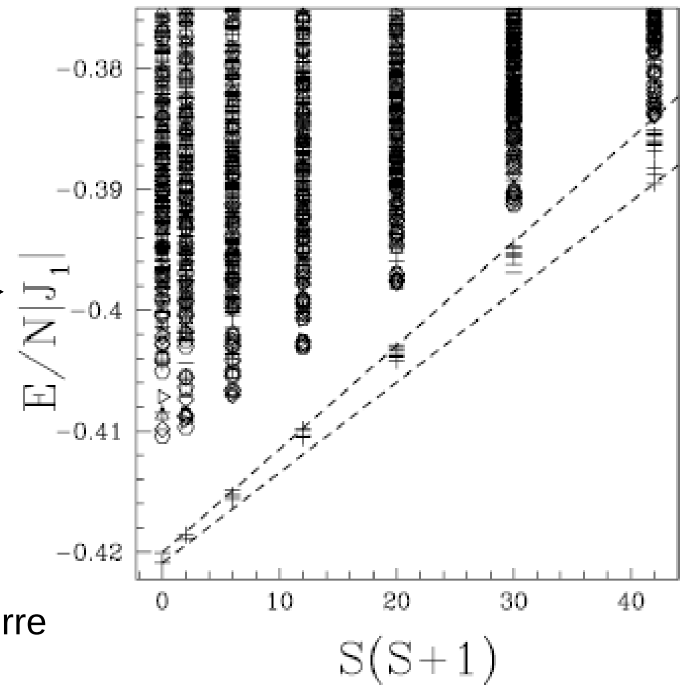
Anderson, PR 1953

Bernu *et al.*, PRL 1992, PRB 1994

Lhuillier, cond-mat/0502464







Example with 12 sublattices !

J_1 - J_2 model on the kagome lattice

J.-C. Domenge, P. Sindzingre, C. Lhuillier and L. Pierre

PRB 2005

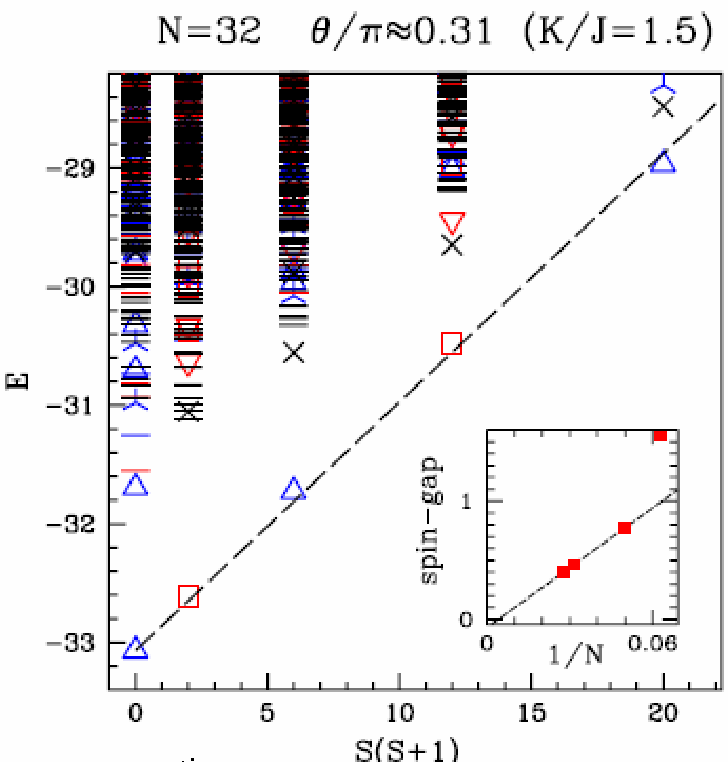
Quantum spin nematics

[Andreev & Grishchuk JETP 1984; Chandra & Coleman PRL 1991]

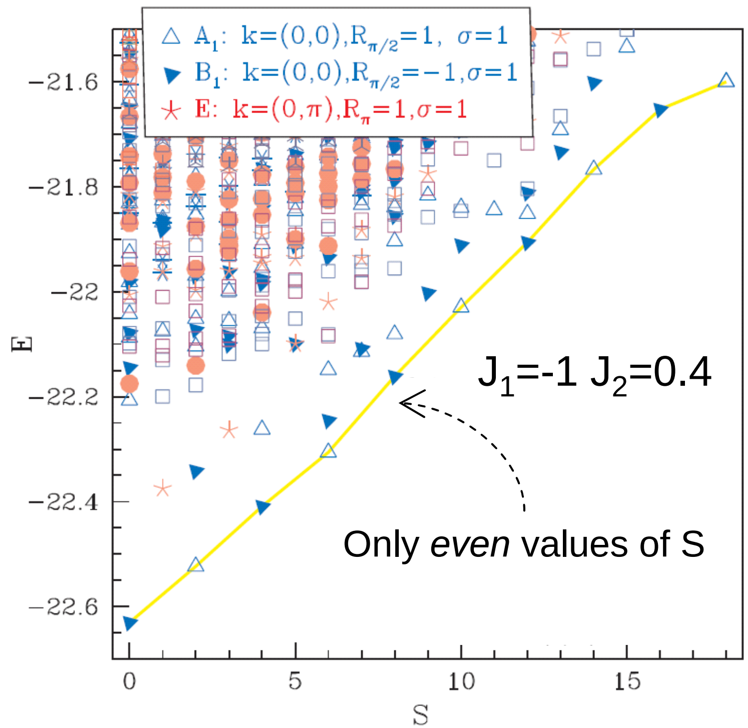
- Spontaneous breakdown of SU(2) symmetry but short-ranged spin-spin correlations. The order parameter involves 2 spins operators

$$\left\langle \vec{S}_r \right\rangle = 0$$

Spontaneous selection of a (oriented or non oriented) *plane*



p-nematic
Spin- ½ model with 4-spin ring exchange (square lattice)
Läuchli *et al.*, PRL 2005



n-nematic
Shannon, Momoi & Sindzingre, PRL 2006

[See Also: Tsunetsugu & Arikawa JPSJ 2006, Läuchli, Penc & Mila, cond-mat/0605234]

Conclusions

- Exact diagonalizations, when coupled to a full symmetry analysis is a powerful tool specially for D>1 quantum frustrated magnets
- The spectrum itself is a rich source of information concerning broken symmetries
- Very useful if only very small systems are numerically available (spin > ½, spin-orbital models, doped magnets, …)
- Complementary approach to obtain the order parameter from the quasi-degenerate wave-functions (using reduced density matrices): Furukawa, GM & Oshikawa PRL 2006