Spinons and gauge degrees of freedom in spin liquids

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“*Two-dimensional quantum antiferromagnets* "
GM and C. Lhuillier, review chapter in the book ``Frustrated spin systems'', edited by H. T. Diep, World-Scientific (2005). [cond-mat/0310405]

Introduction

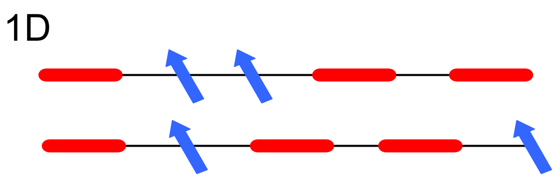
- What is fractionalization ?
  - Excitations with quantum numbers which are fraction of the elementary degrees of freedom.

    Most famous example: q=e/3 in FQHE.
  - In magnetic systems:

    An s=½ spinon (charge neutral) is a "fraction" of an electron.

    (or a *fraction* of a $\Delta S^z=1$ spin flip)
  - Very natural in 1D (domain wall or soliton)

    But more complex in higher dimension…



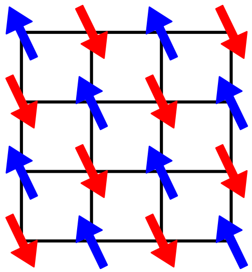
- This talk
  - Attempt to explain simply some (not so recent) theoretical approaches to confinement and deconfinement in magnets

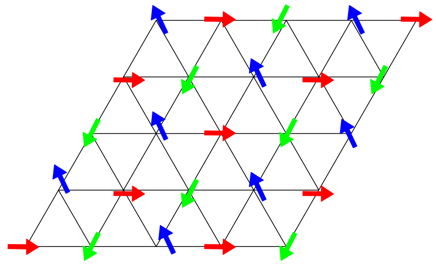
    Natural language: *gauge theory*.
  - Mention some recent spin (or related) models realizing deconfined phases

Spin-½ antiferromagnetic Heisenberg models

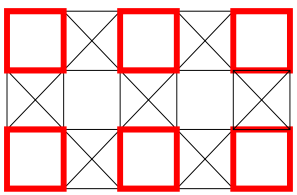
$$H = J\sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + \cdots$$

Many possible phases characterized by different (spontaneously) broken symmetries

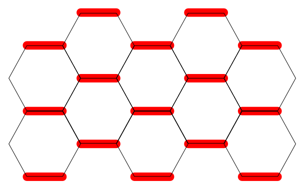




Square, triangular and hexagonal lattices:
Néels states
- Spontaneously broken SU(2) symmetry
- Gapless spin waves (ΔS^z=1)

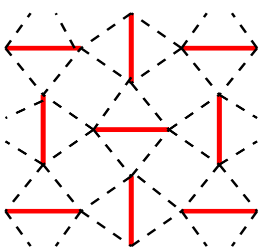


Fouet *et al*. PRB (2003)

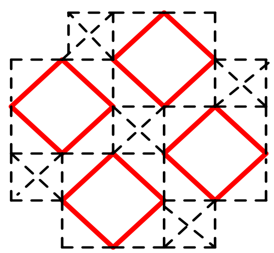


Fouet *et al*. EPJB (2001)

S=0 plaquettes or dimer crystal.
-Spontaneous breakdown of some lattice symmetries
- Gapped magnons (ΔS=1)



$SrCu_2(BO_3)_2$ Kageyama *et al. (1999)*
Shastry-Sutherland model (1981)



CaV_4O_9 Taniguchi *et al.* J. Phys. Soc. Jpn (1995)
$\Delta \approx$100 K - 1st 2D spin-gap system

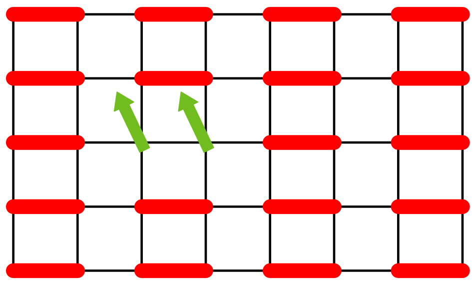
Even number of spins per unit cell:
- Possibility of no broken symmetry
- Gapped magnons (ΔS=1)

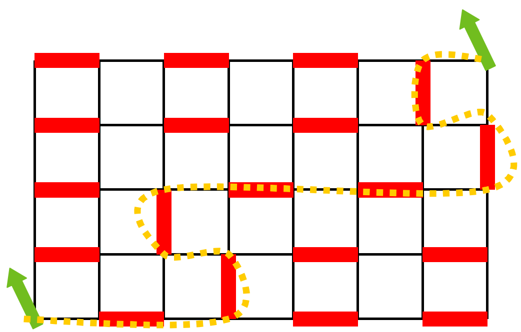
So far no spinon…

Confinement/deconfinement in terms of valence-bonds

Valence-bond crystal

 $\approx \frac{1}{\sqrt{2}}\left(|\uparrow\rangle - |\downarrow\rangle\right)$



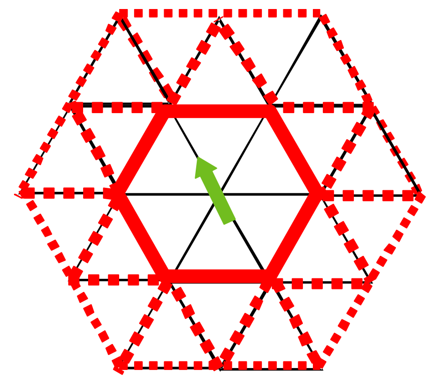


Energy grows linearly with distance
$\Rightarrow$ Confinement

Resonating valence-bond liquid

(short-ranged)
[P. W. Anderson, 1973; 1987]

- No broken symmetry
- Short-ranged correlations only



One spinon is surrounded by a *local* reorganization of the (liquid-like) valence-bond background.

$\Longrightarrow$ (possibility of) Deconfinement

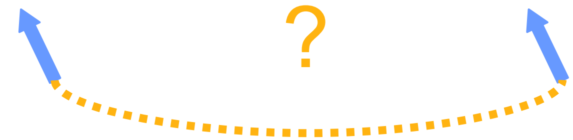
Formalism ? Examples ?

Gauge theory descriptions of spin models

Spin model: Local microscopic model

No explicit long-ranged interaction.

$\Rightarrow$ How to understand confinement / deconfinement ?



Gauge theories provide a natural framework…

Example: particles on a lattice

$$H = \sum_{rr'} t_{rr'} \left[b_r^+ e^{-iA_{rr'}} b_{r'} + \mathrm{H.c} \right] + \cdots$$

Local **redundancy** :

$$\left.\begin{aligned} b_r &\to e^{i\theta(r)} b_r \\ A_{rr'} &\to A_{rr'} + \theta(r) - \theta(r') \end{aligned}\right\}$$

Gauge transformation
(time independent)

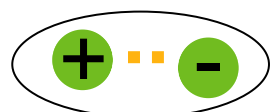
Physical quantum states must be invariant under such gauge transformations

$\vec{E}_{rr'}$: Electric field, conjugate to $A_{rr'}$

$$\left(b_r^{\,+} b_r - \sum_{r'} E_{rr'} \right) |\psi\rangle = 0$$

Local constraint (Gauss law)

+ (isolated) "*charged*" excitations have a finite energy = deconfinement −



Only "*neutral*" excitations
= confinement

Concrete example for a spin model ?

Example of a slave particle representation of the spins operators

- Schwinger boson representation of SU(2) & Heisenberg model with S=1/2

$$\boxed{\begin{array}{l} S^z = \frac{1}{2}\left(b_\uparrow^+ b_\uparrow - b_\downarrow^+ b_\downarrow\right) \\ S^+ = b_\uparrow^+ b_\downarrow \qquad S^- = b_\downarrow^+ b_\uparrow \\ \vec{S}^2 = \frac{1}{2}\left(\frac{1}{2}+1\right) \Rightarrow b_\uparrow^+ b_\uparrow + b_\downarrow^+ b_\downarrow = 1 \end{array}} \quad (S = \tfrac{1}{2})$$

$b_\uparrow^+$ (or $b_\downarrow^+$) creates a spinon

$$\chi_{rr'}{}^+ = \frac{1}{2}\left(b_{\uparrow r}{}^+ b_{\downarrow r'}{}^+ - b_{\downarrow r}{}^+ b_{\uparrow r'}{}^+\right)$$ creates a spin singlet

$$\vec{S}_r \cdot \vec{S}_{r'} = \frac{1}{4} - 2\chi_{rr'}^+ \chi_{rr'}$$

$\theta(r)$ arbitrary angle at each site :

$$\boxed{\left.\begin{array}{c} b_{\sigma=\uparrow,\downarrow r} \to e^{i\theta(r)} b_{\sigma r} \\ \vec{S}_r \to \vec{S}_r \end{array}\right\} \text{ local } U(1) \text{ redundancy}}$$

Review on slave particle approaches to the *t-J* model:
P. A. Lee, N. Nagaosa, X. G. Wen
cond-mat/0410445

Slave particles and U(1) Gauge symmetry - Schwinger bosons (II)

- Formulation in terms of spinons interacting with bond fields $Q_{rr'}$

$$Z = \mathrm{Tr}\left[\exp(-\beta H)\right]$$

$$= \int D\left[\boxed{b_{r\uparrow}, b_{r\downarrow},}\ \boxed{Q_{rr'},}\ \boxed{\lambda_r}\right]\exp(-S_{eff})$$

Lagrange multiplier

$$S_{\mathrm{eff}} = \int d\tau \sum_{rr'} \left\{ \frac{1}{2}\frac{|Q_{rr'}|^2}{J_{rr'}} - \left(Q^+_{rr'} \underbrace{\left(b_{\uparrow r} b_{\downarrow r'} - b_{\downarrow r} b_{\uparrow r'}\right)}_{\chi_{rr'}} + \mathrm{h.c} \right) \right\} + \cdots$$

Bipartite lattice :

$$(-1)^r = \begin{cases} +1 & r \in A \\ -1 & r \in B \end{cases}$$

$$b_{\uparrow r \in A} \to e^{i\theta(r)} b_{\uparrow r} \quad \Rightarrow \text{'electric' charge } +1$$

$$b_{\uparrow r \in B} \to e^{-i\theta(r)} b_{\uparrow r} \quad \Rightarrow \text{'electric' charge } -1$$

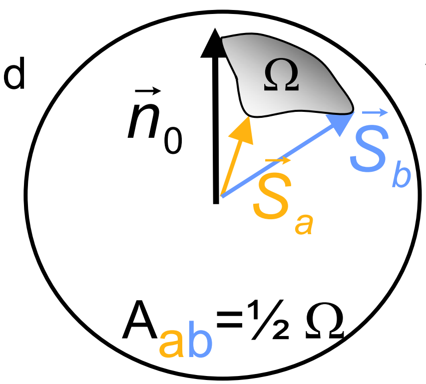
$$\boxed{\arg(Q_{rr'}) = A_{rr'} \to A_{rr'} + \theta(r) - \theta(r')} \Rightarrow \text{compact } U(1) \text{ gauge field}$$

- Geometrical interpretation of the gauge field :

Solid angle defined by 2 spins and a reference $\vec{n}_0$

Gauge transformation $\Rightarrow$ change of reference direction

gauge flux = solid angle density ~ non-collinearity of the spins



What is special with compact gauge fields ?

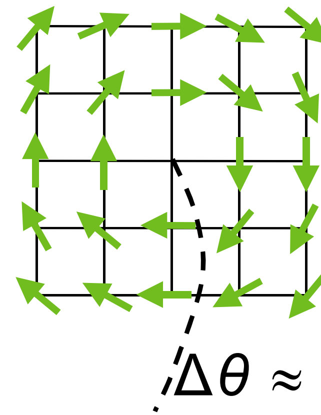
Read and Sachdev PRL 1989; PRL 1991;…

Monopoles in U(1) gauge theory in D=2+1

- Analogy with **vortices** in the 2D classical O(2) model

$$E = \sum_{\langle ij \rangle} \left[1 - \cos(\theta_i - \theta_j) \right]$$

$$\theta_i \in \left] -\pi, \pi \right]$$

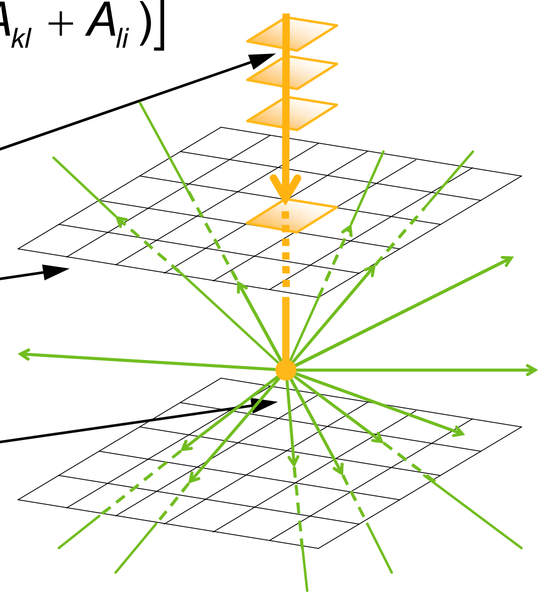


Energy $\approx \log(L)$

$\Delta\theta \approx 2\pi$

- Magnetic **monopoles** in a U(1) gauge theory

$$S = \sum_{\langle ijkl \rangle} \left[1 - \cos(A_{ij} + A_{jk} + A_{kl} + A_{li}) \right]$$



Time axis

$$\sum_r B(r, t_2) = \varphi_0 + 2\pi$$

$$\sum_r B(r, t_1) = \varphi_0$$

Finite action

$S \approx O(1)$

[Polyakov 1975; 1977; 1987;
Fradkin & Susskind 1979]

Consequences ?

Phases of U(1) Gauge theories in D=2+1 dimensions

What this known about such U(1) gauge theories ?

- monopoles proliferate [Polyakov 1975; 1977; 1987]

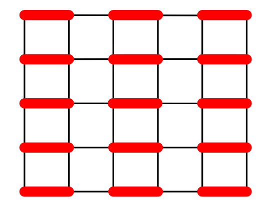
  $\Rightarrow$ Confinement.

  The spinons are glued in pairs by the strong gauge-field fluctuations and are not physical excitations ☹…

For a S=½ system, monopoles have non-trivial *Berry phases*
[Haldane PRL 1988; Read & Sachdev PRL 1989]

Analogy with the topological term which makes the difference between 2S *odd* and *even* in spin chains [Haldane 1983].

$\Rightarrow$ The confined phase is a valence-bond crystal



- Deconfinement possible in presence of gapless matter fields

  So called *U(1) spin liquid* [Affleck & Marston 1988, …, Hermele *et al.* PRB 2004]

- In presence of a charge-2 field the U(1) 'symmetry' can be broken down to Z_2, leading to deconfinement [Fradkin & Shenker, PRD 1979]

How can this happen in a spin system ?

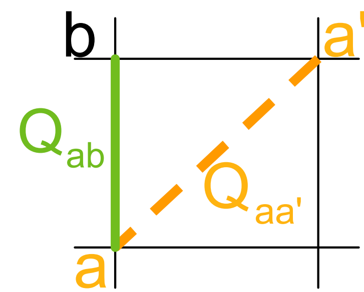
From a U(1) to Z_2 gauge theory

Read and Sachdev PRL 1991

$$S_{\text{eff}} = \int d\tau \sum_{rr'} \left\{ \frac{1}{2}\frac{|Q_{rr'}|^2}{J_{rr'}} - \left(Q^{+}_{rr'} \underbrace{\left(b_{\uparrow r} b_{\downarrow r'} - b_{\downarrow r} b_{\uparrow r'}\right)}_{\chi_{rr'}} + \text{h.c} \right) \right\} + \cdots$$

$$Q_{ab} \to e^{i\boxed{(\theta(a)-\theta(b))}} Q_{ab} \Rightarrow \text{'electric' charge } 0$$

$$Q_{aa'} \to e^{i\boxed{(\theta(a)+\theta(a'))}} Q_{aa'} \Rightarrow \text{'electric' charge } 2$$



Non-collinear spin-spin correlations

$\Rightarrow$ Some bond variables $Q_{aa'}$ connecting 2 sites on the *same* sublattice may acquire a finite expectation value

$$Q_{aa'} \to Q_{aa'} e^{2i\theta} \quad \underbrace{\langle Q_{aa'} \rangle \neq 0}_{\text{mean-field}} \Rightarrow \text{U(1) redundancy broken to } Z_2 : \theta \in \{0, \pi\}$$

(Condensation of a charged particle $\Rightarrow$ Anderson-Higgs mechanism, Meissner effect)

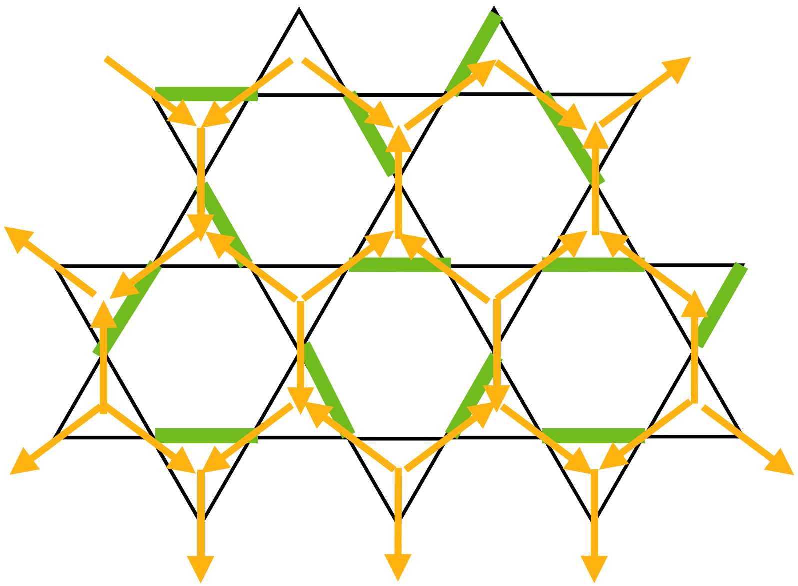
Z_2 gauge theories [$A_{rr'}$=0 or π] *do have a deconfined phase* ☺

A concrete example ?

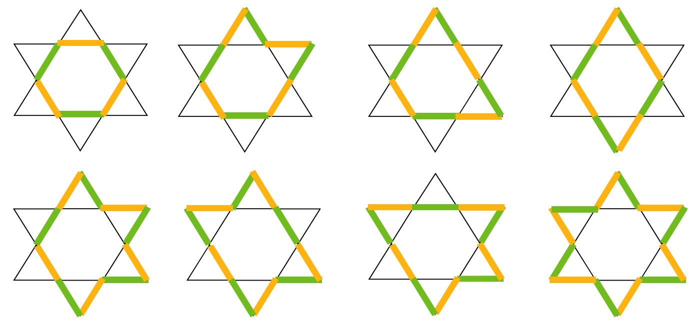
Solvable dimer (toy) model realizing a Z_2 liquid (I)

[GM, Serban & Pasquier PRL 2002]

- Arrow representation of dimer coverings on the kagome lattice



Constraint
Number of incoming arrows must be *even* on every triangle



- Hamiltonian

$\tau^z(i) =$ Flips the arrow $\boldsymbol{i}$

$$H = -\sum_h \underbrace{\prod_{i=1}^{6} \tau_i^z}_{\text{dimer hoping}}$$

Where is the Z_2 gauge theory ?

Solvable dimer (toy) model realizing a Z_2 liquid (II)

[GM, Serban & Pasquier PRL 2002]

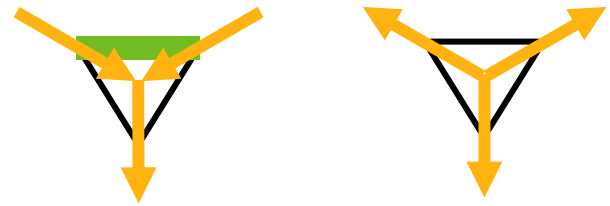
- Arrow = Z_2 Gauge field

Gauge field

$$\tau^z(i) = \text{Flips the arrow } \boldsymbol{i} = e^{iA_{rr'}}$$

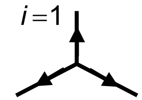
Electric field

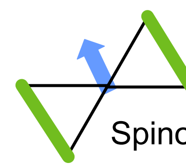
$$\tau^x(i) = \begin{cases} +1 & \text{If the arrow } \boldsymbol{i} \text{ is the same as in some reference configuration} \\ -1 & \text{Otherwise} \end{cases}$$



- Constrain= Gauss law

$$\prod_{i=1}^{3} \tau_i^x = 1 \Leftrightarrow \text{div } \vec{E} = 0$$





- Hamiltonian = magnetic energy

$$H = -\sum_h \underbrace{\prod_{i=1}^{6} \tau_i^z}_{\substack{\text{Magnetic field} \\ \exp[\, iB(h)\,]=\pm 1}} \cancel{-\Gamma \sum_i \underbrace{\tau_i^x}_{\text{Electric field}}}$$

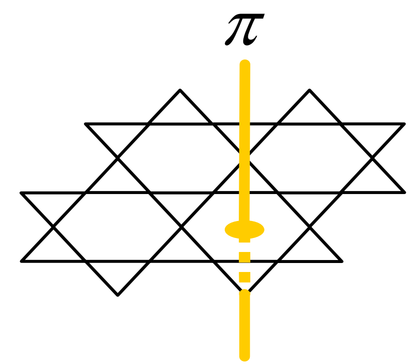
- Ground-state: uniform dimer liquid

$$\forall h \quad B(h)\left|0\right\rangle = 0$$

Not that trivial in terms of the original dimers…

What is a deconfined Z_2 state ?

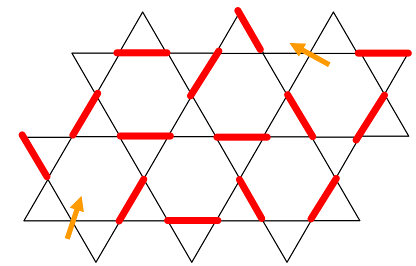
- No broken symmetry
  - No long-ranged correlations
  - No local order parameter
  - Short-ranged RVB state: dimer~spin singlet



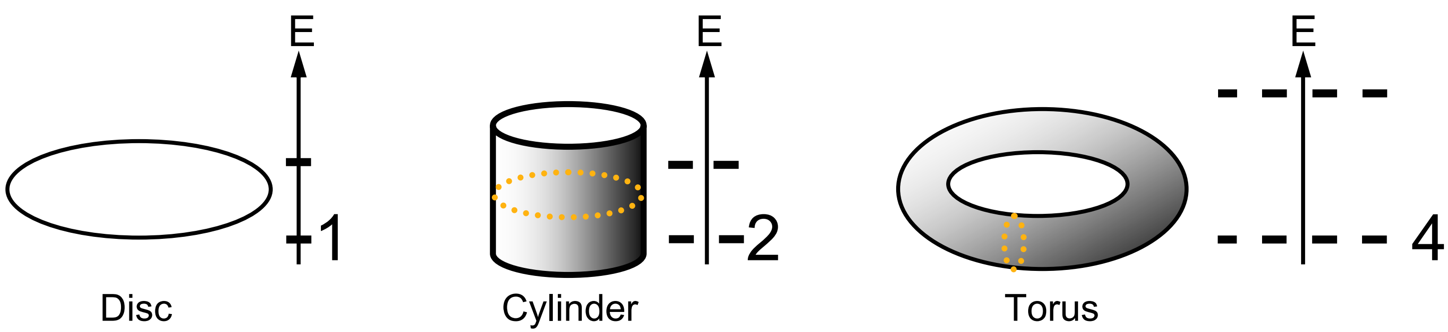
- Gapped excitations

  = Elementary flux (vortex) of a Z_2 gauge theory = *visons*

  [Read & Chakraborty PRB 1989; Kivelson PRB 1989;
  G. X. Wen PRB 1991; Senthil & Fisher PRL 2001]



- Deconfined fractional excitations (spinons)
- Topological degeneracy – topological order [G. X. Wen PRB 1991]



Ground-states are *locally* indistinguishable.

Degeneracy *robust* to any local *perturbation*.

[Furukawa, GM, Oshikawa (unpublished); GM, Pasquier, Lhuillier & Mila PRB 2005]

Other examples ?

Examples of deconfined Z_2 Liquids (I)

- **Heisenberg-like models** (SU(2) or larger continuous symmetry)
  - Large-*N* frustrated antiferromagnets with Sp(N) symmetry [Read & Sachdev PRL 1991]
  - Multiple-spin exchange model on the triangular lattice ?

    Exact diagonalization study [GM, Lhuillier, Bernu, Waldtmann PRB 1999]
  - J_1-J_2 model on the honeycomb lattice ? [Fouet *et al.* PRB 2003]

    square lattice ? [Capriotti *et al.* PRL 2001]
  - Perturbed Klein models [S. Fujimoto PRB 2005; Raman, Moessner & Sondhi PRB 2005]
  - $CsCuCl_3$: Spinon-like continuum observed with inelastic neutron scattering.

    [Coldea, Tennant *et al.* PRL 2001; PRB 2003]
- **Ising-like models**
  - Ising like model with multiple-spin interactions.

    [Kitaev, cond-mat 1997], [Nayak & Shtengel PRB 2001] [X. G. Wen PRL 2003]
  - J_1-J_2-J_3 Heisenberg model on the kagome lattice with easy axis (Ising) anisotropy:

    [Balents, Fisher & Girvin PRB 2002; D. N. Sheng and Balents 2004]

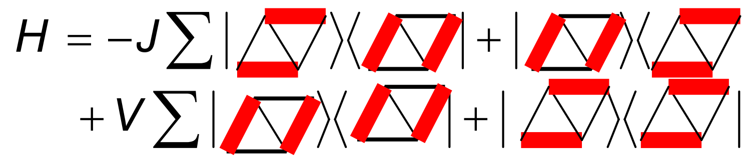
Examples of deconfined Z_2 Liquids (II)

Quantum dimer models

Effective description of singlet valence-bond dynamics

- Triangular lattice

  [Moessner & Sondhi PRL (2001)]



- Kagome lattice

  Completely solvable ! [GM, Serban & Pasquier PRL (2002)]

- 3D non-bipartite lattices (fcc,…)

  [Moessner & Sondhi PRB (2003)]

Other

- Josephson junction arrays

  [Ioffe *et al.*; Douçot, Fegeil'man & Ioffe PRL (2003)]

- Bose-Hubbard models

  [Senthil & Motrunich PRB (2002); PRL (2002)]

- Classification with Projective Symmetry Groups [X. G. Wen PRB 2002]

« Coulomb phase »

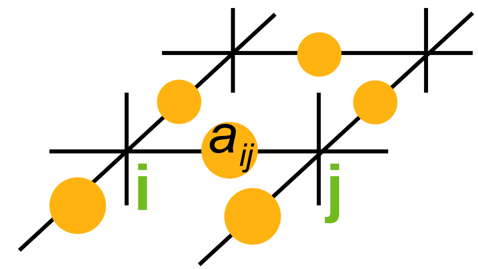
- **A phase *without* isolated magnetic monopoles**
  Analogy with the 2D XY model
  at low temperature where vortices are bound in pairs.

- **Similar to conventional QED:**
  - Deconfined spinons (“electric” charges) with 1/r interaction
  - Gapped “magnetic” monopoles (also with 1/r interaction)
  - Gapless transverse excitation,
    = “photon” with linear dispersion relation $\omega(k)=c|k|$

  BUT NO SPONTANEOUSLY BROKEN SYMMMETRY !
  - Algebraic correlations.

- **Some microscopic models**
  - Bose-Hubbard models
    [Motrunich & Senthil PRL (2002); PRB (2005); Wen PRB (2003)]
  - Quantum dimer model on the cubic lattice
    [Huse *et al.* PRL 2003; Moessner & Sondhi PRB (2003)]
  - S=1/2 model on the pyrochlore lattice
    with strong Ising anisotropy
    [Hermele, Fisher & Balents PRB (2004)]
  - SU(2) spin models [Raman, Moessner & Sondhi PRB 2005]

Quantum rotors on the links of a cubic lattice



$$b_{ij}^{+} = \exp(i a_{ij})$$

Phase of the boson creation operator ⇔ Gauge field

Boson number ⇔ Electric field

Number of bosons touching a given site ⇔ div E

$$H^{eff} = \alpha \sum_{\langle ij \rangle} \vec{E}^2 + \beta \sum_{\langle ijkl \rangle} \vec{B}^2$$

Conclusions

- Fractionalization in frustrated magnets exists !

  Several microscopic models are now available

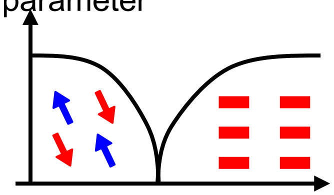
  No clear experimental evidence in D>1 so far…

- U(1) and Z_2 gauge theories provide a natural language to describe these "exotic" phases.

  - Relation to topological order [see also Oshikawa & Senthil cond-mat/0506008]
  - Gauge excitations (and even fermionic excitations)

- Recent developments :
  - "String-net condensation" picture [X. G. Wen 2002-2005] to explain the origin of gauge degrees of freedom
  - "Deconfined critical points" [Senthil *et al.* Science (2004)]

    New kind of continuous quantum phase transitions between phases with different broken symmetries. Fractional excitations *at* the critical point. Role of gauge theory description, topological defects.



  - "Application" to topological quantum bits ?

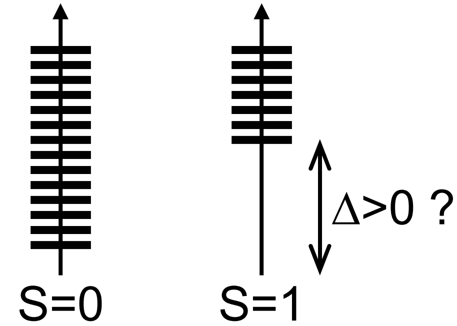
What about the kagome spin-½ AF Heisenberg model ?

- Spin liquid with gapless singlet excitations ?

  No magnetic long-ranged order (almost surely)

  Triplet probably gapped

  [Waldtmann *et al.* Eur Phys. J. B (1998) + Refs. therein]



- Maybe deconfined

  [Dommange *et al.* PRB (2003); Läuchli & Poilblanc PRL (2004)]

  see also [GM, Serban & Pasquier PRB (2003), J. Phys Cond. Mat. (2004)]

- Should be a Z_2 liquid according to Sp(N) approach…

  [Sachdev PRB 1992]

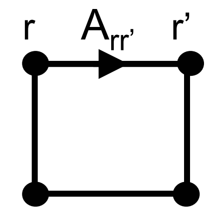
- Might also be a valence-bond crystal…

  [Marston & Zeng J. App. Phys. 1991; Nikolic & Senthil PRB (2003)]

Latttice gauge theory

(Abelian)

Continuum | Lattice



Continuum:

$$H=\frac{1}{2m}\left(\vec{p}-q\vec{A}\right)^2+\cdots$$

Lattice:

$$H=\sum_{rr'} t_{rr'}\left[b_r^+ e^{-iA_{rr'}} b_{r'} + \mathrm{H.c}\right]+\cdots$$

$$A_{rr'}\in\begin{cases}[-\pi,\pi[\Rightarrow U(1)\text{ gauge theory}\\ \{0,\pi\}\Rightarrow Z_2\text{ gauge theory}\end{cases}$$

Continuum:

$$\psi(r)\to e^{iq\theta(r)}\psi(r)$$
$$\vec{A}\to\vec{A}+\vec{\nabla}\theta$$

Lattice:

$$\left.\begin{array}{l} b_r \to e^{i\theta(r)} b_r \\ A_{rr'}\to A_{rr'}+\theta(r)-\theta(r')\end{array}\right\}$$

Gauge transformation
(time independent)

Continuum:

$$\vec{B}=\vec{\nabla}\times\vec{A}$$

Lattice:

$$B=\sum_{\square} A_{rr'}$$

Continuum:

$$\vec{E}=-\vec{\nabla}A_0-\partial_t\vec{A}$$

Lattice:

$\vec{E}_{rr'}$: Electric field, conjugate to $A_{rr'}$

Continuum:

Gauss law div $\vec{E}=\rho$

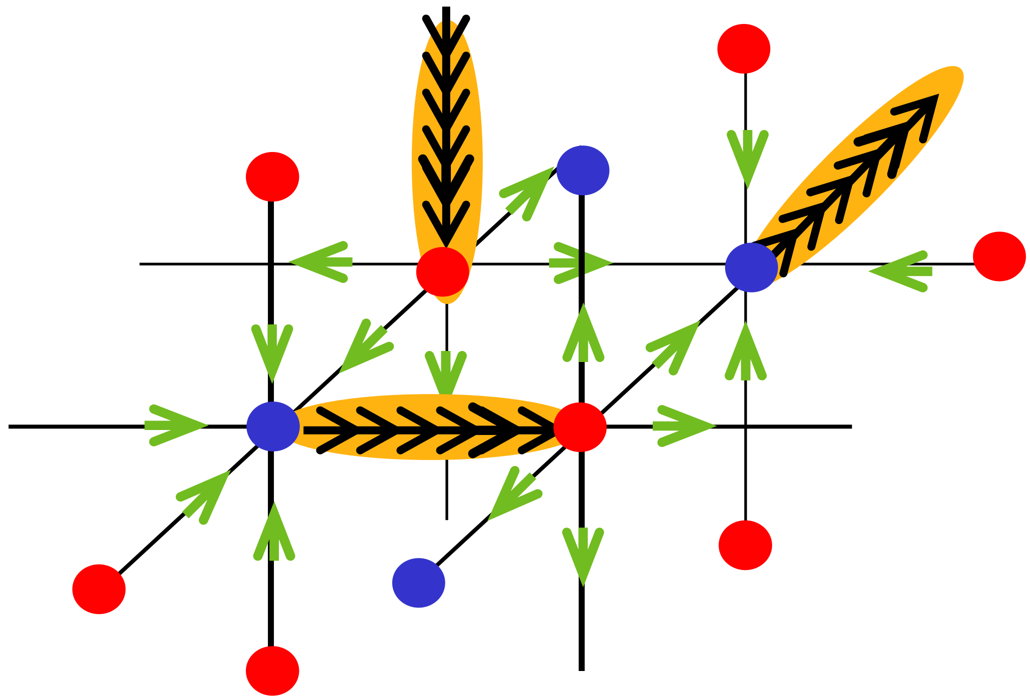
Lattice:

$$\left(b_r^+ b_r-\sum_{r'}E_{rr'}\right)\left|\psi\right\rangle=0$$

Local constraint

Quantum dimer models in 3D & Coulomb phase

[Huse *et al.* PRL 2003; Moessner & Sondhi, PRB (2003)]



Bi-partite lattice: sublattices A & B
Dimer A→ *B* = 5 units of electric field
No dimer = -1 unit of electric field
One dimer per site ⇒ $\mathrm{div}\ \vec{E} = 0$