Quantum dimer model with a liquid ground-state: topological degeneracy and toy model for a topological quantum-bit

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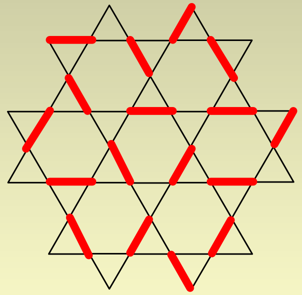
soon on cond-mat/0410...

Outline

- What is a quantum dimer model ?
- What is a Z_2 liquid ? Topological degeneracy ?
- What is a topological quantum bit ?



- Why studying QDM's on the *kagome* lattice ?
  - Very simple: mapping onto (transverse field) Ising models
  - Exact Z_2 liquid ground-state, topological degeneracy , etc.

- How to lift a topological degeneracy ?
  - with "scissors" to change the topology
  - with monomers to mix different sectors

- Quantum-bit manipulation
  - Rotation, projection…

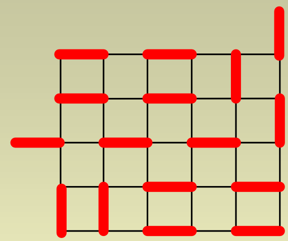
What is a quantum dimer model ?

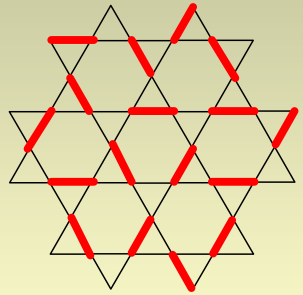
Rokhsar & Kivelson PRL '88

- Fully-packed dimer covering

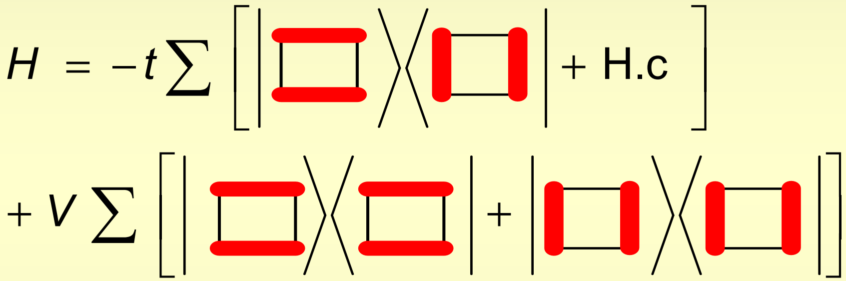
  Several connections exist between QDM & frustrated magnets.

  Example: dimer ~ spin singlet

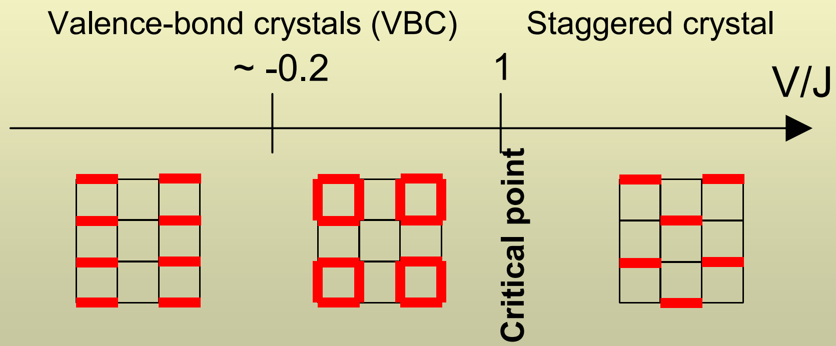




- Introduce some (simple) quantum dynamics



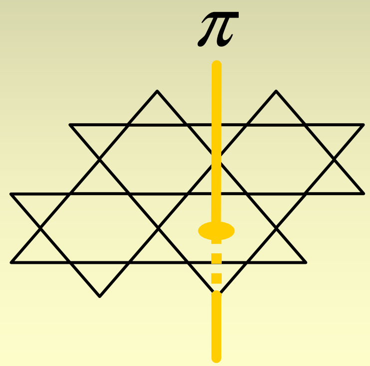
- Example of phase diagram (T=0)



No liquid so far…

What is a Z_2 liquid ?

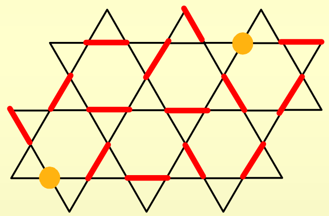
- No broken symmetry
  - No long-ranged correlations
  - No local order parameter
  - Short-ranged RVB state: dimer~spin singlet



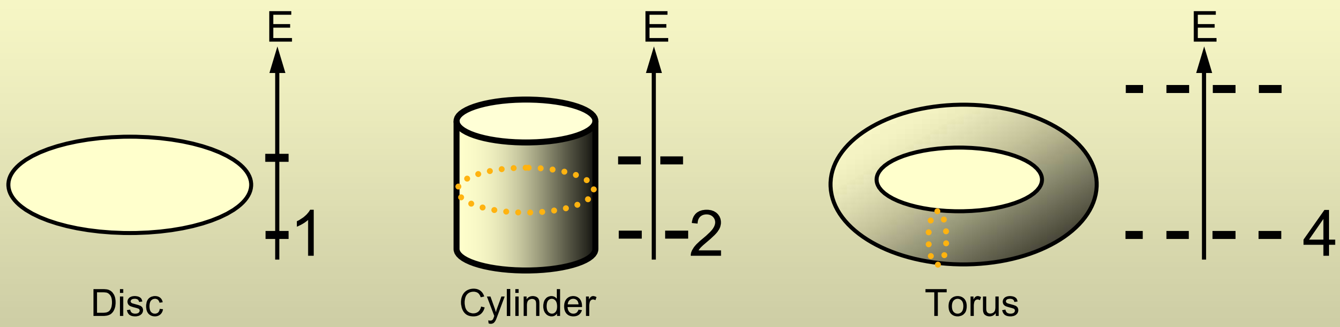
- Gapped excitations

  = Elementary flux (vortex) of a Z_2 gauge theory = *visons*
  [Read & Chakraborty, PRB 1989; Kivelson, PRB 1989; G. X. Wen PRB 1991; Senthil & Fisher PRL 2001]

- Deconfined fractional excitations (monomers for instance)



- Topological degeneracy – topological order



Ground-states are *locally* indistinguishable.

Degeneracy *robust* to local *perturbations*.

Can this be used for something ?

What is a topological quantum bit ?

- Qubit = 2-level system

  Used to store/process some quantum information

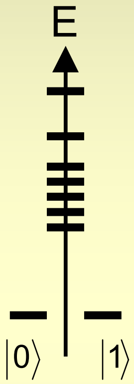
$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

- Topological qubit:

  |0> and |1> are the ground-states of a macro (meso?)scopic system

  which are degenerate because of the (non-trivial) topology.

  Example: **Z_2 liquid on a cylinder.**



- Advantage:

  The two states are *locally* indistinguishable

  $\Rightarrow$ no local perturbation can introduce decoherence.
  - A. Yu. Kitaev, Annals Phys. 303, 2 (2003) [quant-ph/9707021]
  - Ioffe, Feigel'man, Ioselevich, Ivanov, Troyer, and Blatter, Nature **415**, 503 (2002).

- Problems:

  How can it be initialized, manipulated and read ?

Are there some examples of such Z_2 liquids ?

Examples of QDM with Z_2-liquid ground-states

- Triangular lattice

  [Moessner & Sondhi, PRL (2001)]

- Kagome lattice

  [GM, Serban, Pasquier, PRL (2002)]

- 3D non-bipartite lattices (fcc,…)

  [Moessner & Sondhi, PRB (2003)]

- …
  - Heisenberg-like models

    [Sp(N): Read & Sachdev PRL (1991)]
    [Several candidates among 2D frustrated S=½ models: exact diagonalizations studies in C. Lhuillier's group]
  - Ising-like models

    [Nayak & Shtengel PRB (2001)]
    [Balents, Fisher & Girvin PRB (2002)]
  - Bose-Hubbard models

    [Senthil & Motrunich PRB (2002); PRL (2002)]
  - Josephson junction arrays

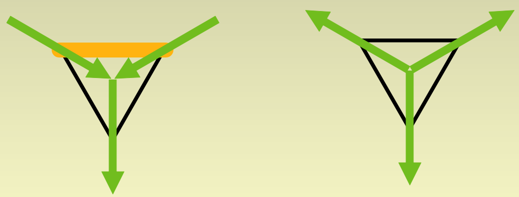
    [Ioffe *et al.*; Douçot, Fegeil'man & Ioffe PRL (2003)]

Let's look at the simplest example

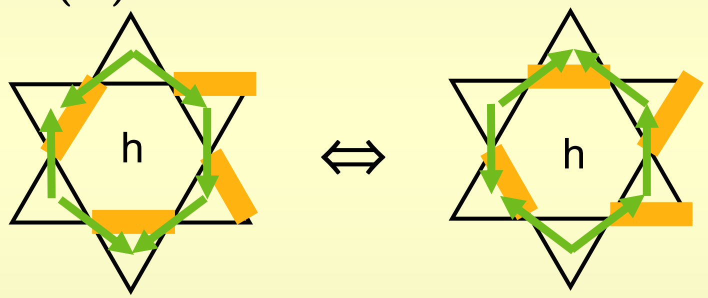
A solvable QDM with Z_2 liquid ground-state

[GM, Serban, Pasquier, PRL (2002)]

On a lattice made of corner-sharing triangles (such as kagome), dimer coverings are easily represented with *arrows*:



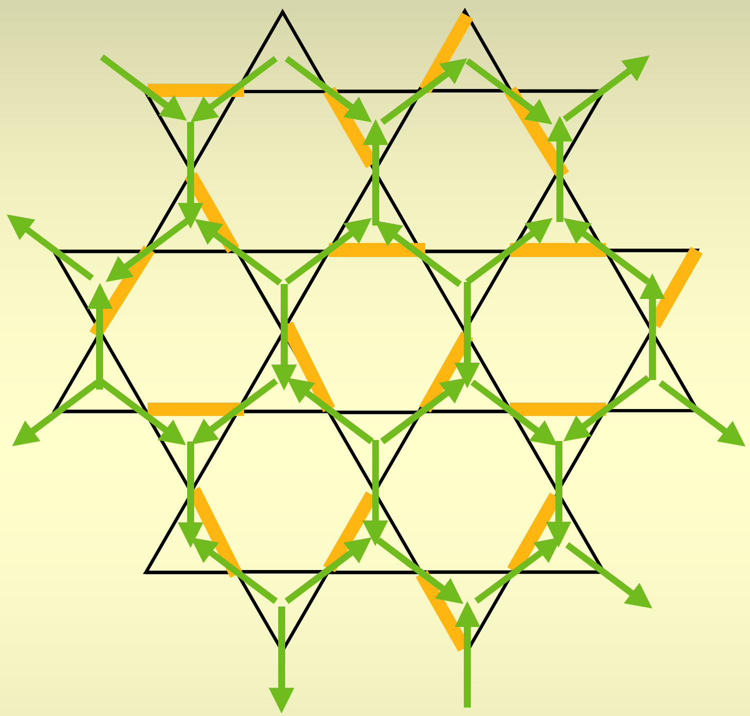
$\sigma^x(h)$: Flips the 6 arrows around h



$$\sigma^x(h)^2 = 1$$

$$[\sigma^x(h), \sigma^x(h')] = 0 \quad \forall h, h'$$

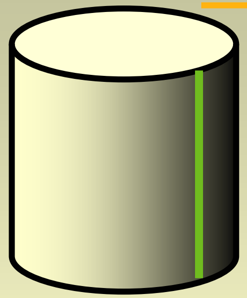
σ^x ⇔ Ising pseudo-spin operator

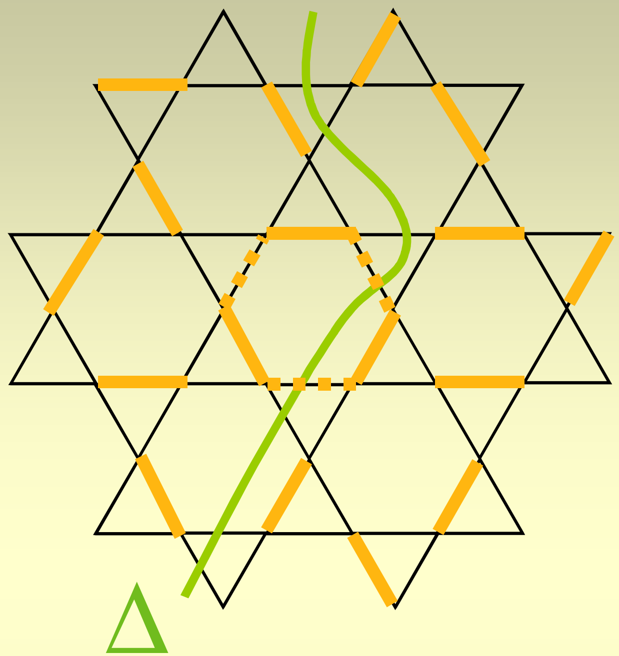


$$H = -\sum_{h \in \text{hexagons}} \sigma^x(h)$$

Where is the topological degeneracy ?

Topological sectors & topological degeneracy





$$T^x = (-1)^{N_\Delta}$$

Dimer number **parity** is conserved by any local dimer move
⇒2 topological sectors

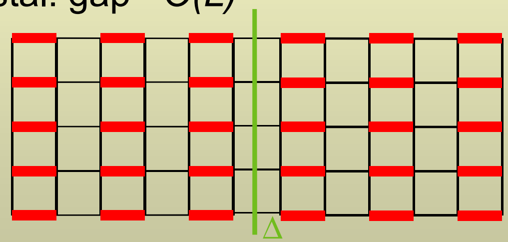
A QDM can be diagonalized separately in each topological sector.

A dimer liquid has the *same ground-state energy in all topological sectors* (in the thermodynamic limit).

Bu in a crystal: gap ~*O(L)*

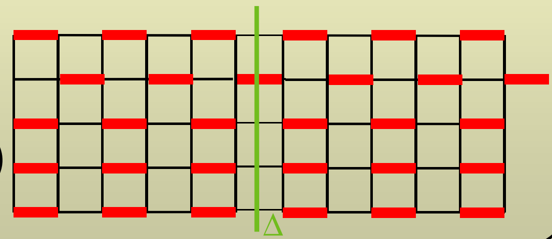
$T^x = 1$

$E_+ = 0$



$T^x = -1$

$E_- \sim O(L)$



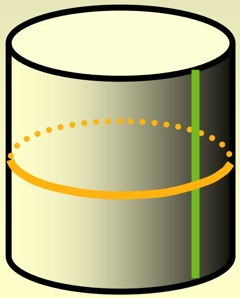
Ground-state degeneracy

τ_i^z = Flips the arrow ***i***

$$\Rightarrow \quad H = -\sum_h \sigma^x(h) = -\sum_h \left(\prod_{i=1 \,(\text{hexagon})}^{6} \tau_i^z \right)$$

$$T^z = \prod_{i \in \Delta^*} \tau_i^z$$

= Shifts all the dimers along Δ^*

Δ^*  Δ

$$T^x = (-1)^{N_\Delta}$$

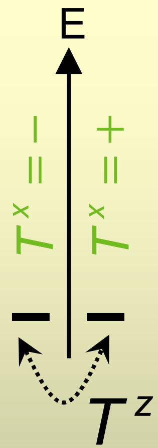
$$[H, T^z] = 0$$

$$[H, T^x] = 0$$

$$T^x T^z = -T^z T^x$$

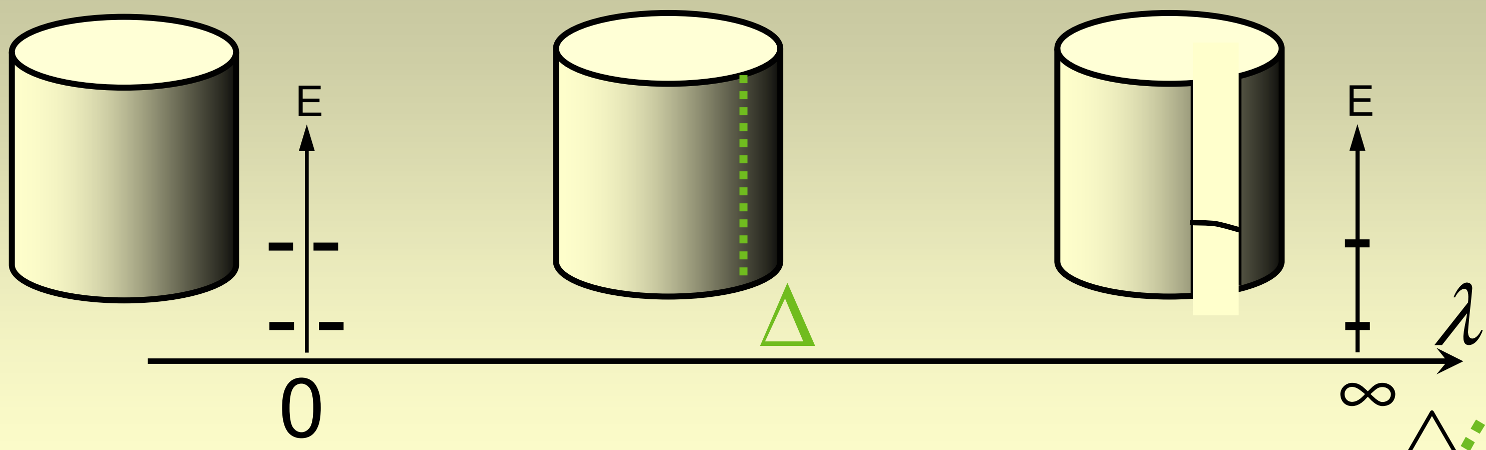
Dimer **shift** — Dimer number **parity**

$\Rightarrow$



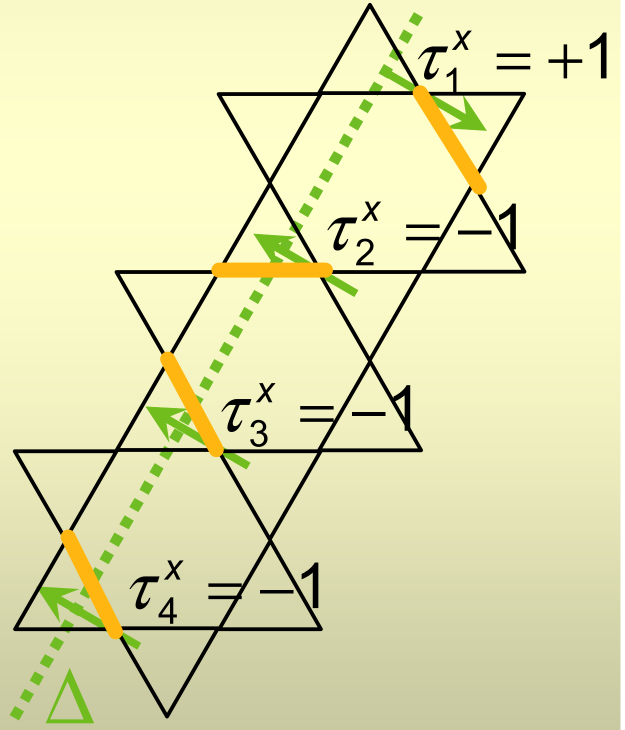
Is this degeneracy robust ? What could lift it ?

From a cylinder to a rectangle



$$H_\lambda = -\sum_{h\in\text{hexagons}} \sigma^x(h) + \underbrace{2\lambda N_\Delta}$$

$$= -\underbrace{\sum_{h\notin\Delta}\sigma^x(h)}_{H_{\text{bulk}}} \underbrace{-\sum_{h\in\Delta}\sigma^x(h) + \lambda \overbrace{\sum_{i=1}^{L}\left(1-\tau_i^x\right)}}_{H_{\text{chain}}(\lambda)}$$



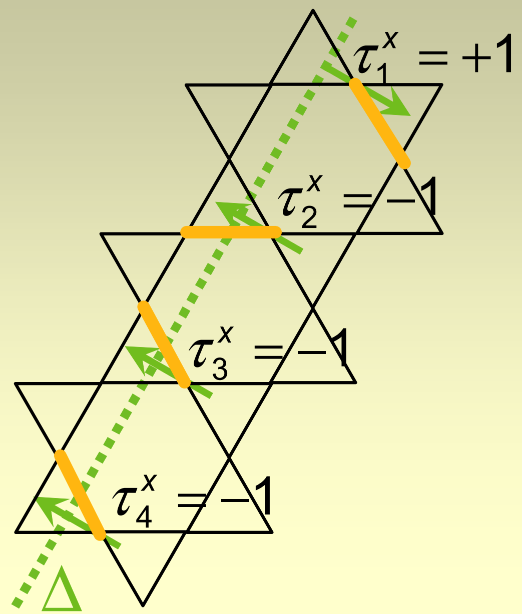
Dimer number parity $T^x = (-1)^{N_\Delta}$ is still a conserved but $\left[H_\lambda, T^z\right] \neq 0$

This model is solvable !

Ising chain in transverse field

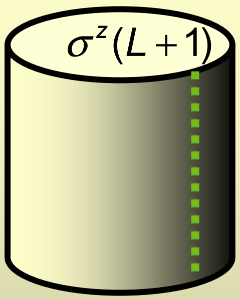
$$H_{\text{chain}}(\lambda) = -\sum_{h\in\Delta}\sigma^x(h) + \lambda\sum_{i=1}^{L}(1-\tau_i^x)$$

$$= -\sum_{h=1}^{L}\sigma^x(h) - \lambda\sum_{i=0}^{L}\overbrace{\sigma^z(i)\sigma^z(i+1)}$$

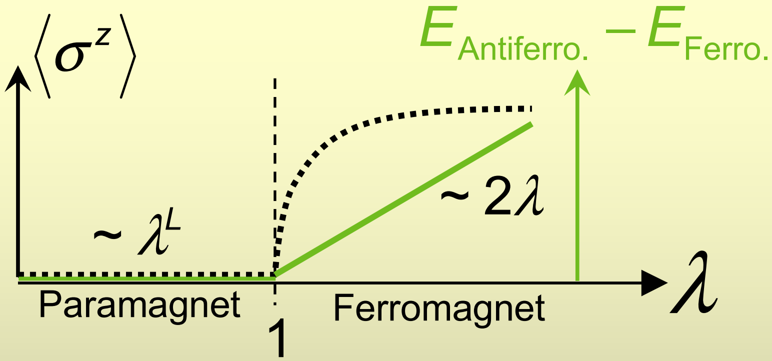


Topological sector coded by
the **boundary conditions of the Ising chain**

$$T^x = (-1)^{N_\Delta} = \prod_{i=1}^{L}\tau_i^x$$



$$= \sigma^z(0)\sigma^z(L+1) = \begin{cases} +1 & \text{ferro.} \\ -1 & \text{antiferro.} \end{cases}$$



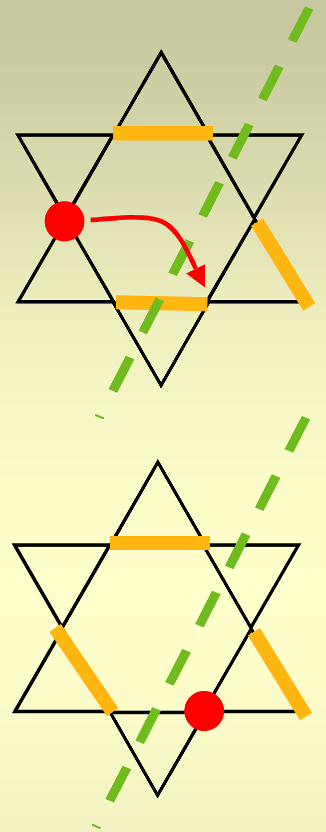
$\rightarrow\rightarrow\rightarrow\rightarrow$ $\uparrow\uparrow\downarrow\downarrow$ $T^x = -1$

$\uparrow\uparrow\uparrow\uparrow$ $T^x = 1$

NB: σ^Z = vison creation operator

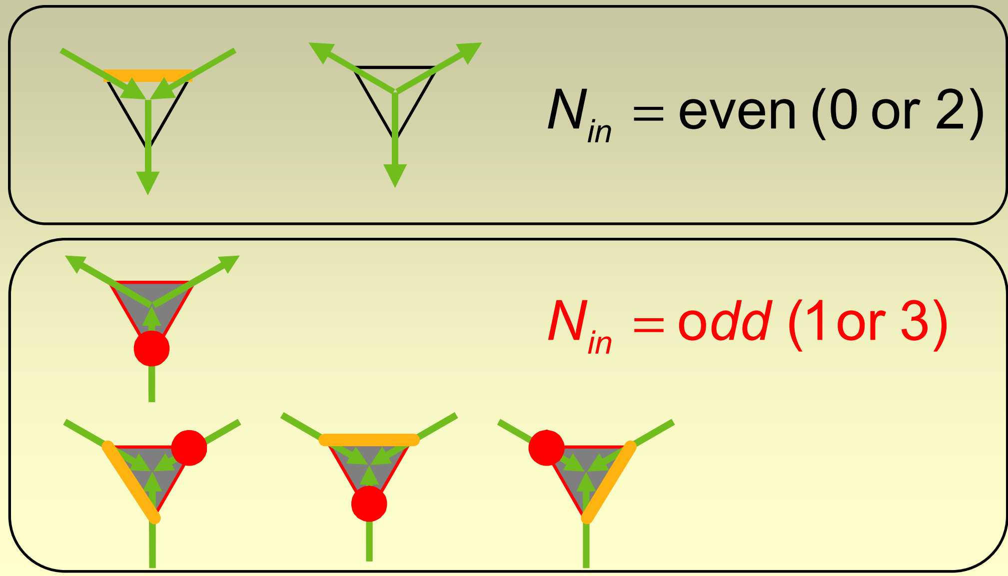
Possibility to *mix* the topological sectors ?

Monomers to mix the topological sectors



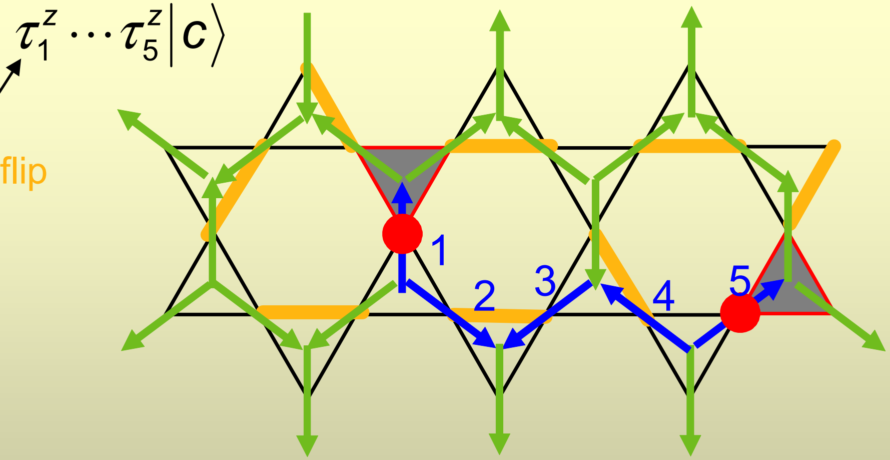
$$T^x = (-1)^{N_\Delta}$$

The dimer number parity is **no longer conserved** in presence of mobile monomers



N_{in} = even (0 or 2)

N_{in} = odd (1 or 3)



Solvable model with monomers ?

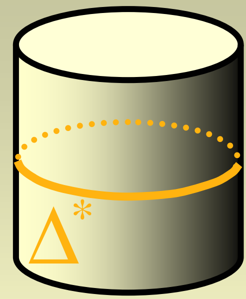
QDM with monomers (I)

$$H = \underbrace{-\sum_h \sigma^x(h)}_{\text{dimer kinetic energy}} \quad \underbrace{-\mu U \sum_{i\in\Delta^*} \tau_i^z}_{\text{monomer pair creation/annihilation \& monomer hopping}} \quad \underbrace{-U \sum_{t\in\text{triangles}} (-1)^{N_{in}(t)}}_{\to \text{constraint } N_{in}=\text{even}}$$

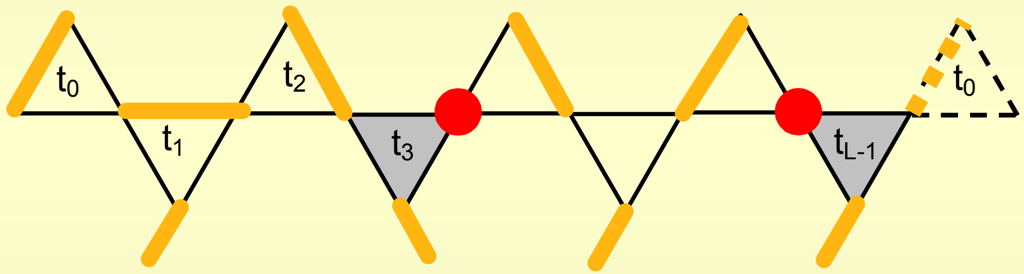
dimer kinetic energy

monomer **pair** creation/annihilation & monomer hopping

$\rightarrow$ constraint N_{in} =even



$$= H_{\text{bulk}} + H_{\text{chain}}(\mu)$$



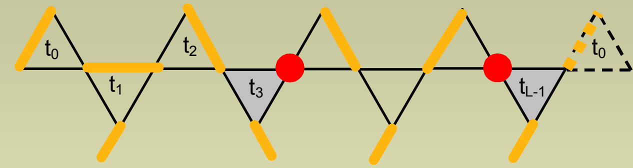
$$H_{\text{chain}}(\mu) = -U \sum_{t=0\cdots L-1} (-1)^{N_{in}(t)} - \mu U \sum_{i=0\cdots L-1} \tau_i^z$$

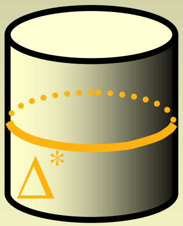
$$= -U \sum_{t=0\cdots L-1} \tilde{\sigma}^x(t) - \mu U \sum_{t=0\cdots L-1} \tilde{\sigma}^z(t) \underbrace{\tilde{\sigma}^z(t+1)}_{\text{monomer creation/annihilation}}$$

monomer creation/annihilation

Ising chain in transverse field

QDM with monomers (II)



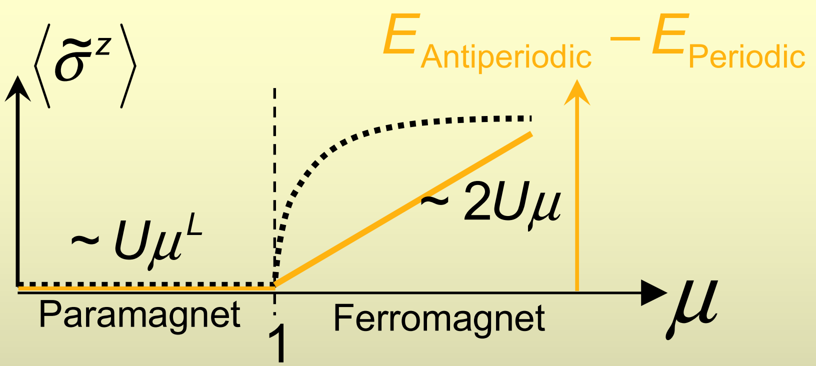


$$H_{\text{chain}} = -U \sum_{t=0\cdots L-1} (-1)^{N_{in}(t)} - \mu U \sum_{i=0\cdots L-1} \tau_i^z$$

$$= -U \sum_{t=0\cdots L-1} \tilde{\sigma}^x(t) - \mu U \sum_{t=0\cdots L-1} \tilde{\sigma}^z(t) \underbrace{\tilde{\sigma}^z(t+1)}_{\text{monomer creation/anihilation}}$$

$T^z = \prod_{i \in \Delta^*} \tau_i^z$ is conserved. Boundary conditions: $\tilde{\sigma}^z(L) = T^z \tilde{\sigma}^z(0)$

$\Rightarrow$ Periodic/Antiperiodic boundary conditions for the Ising pseudospins.

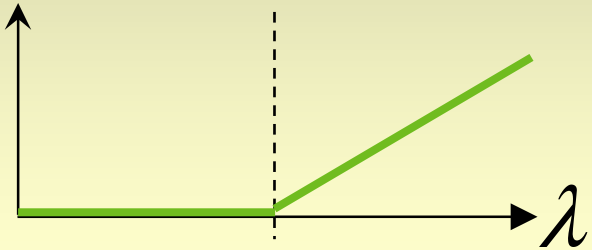


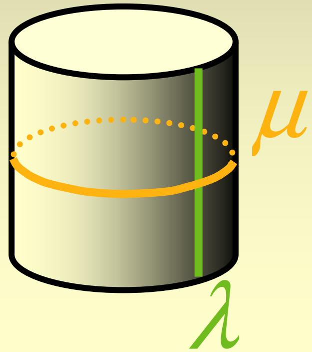
$\rightarrow\rightarrow\rightarrow\rightarrow$ $\quad$ $\uparrow\uparrow\downarrow\downarrow \quad T^z = -1$

$\uparrow\uparrow\uparrow\uparrow \quad T^z = 1$

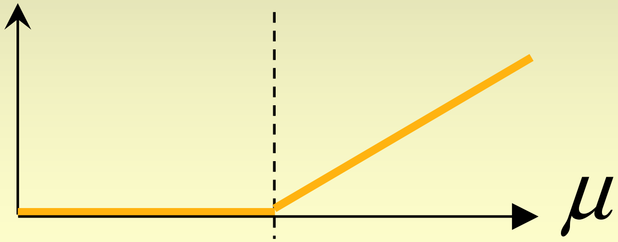
Quantum bit manipulation (I)

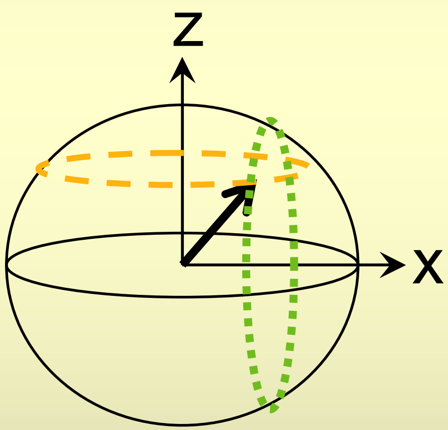
$$f(\lambda) = E_{T^x=-1} - E_{T^x=+1}$$





$$g(\mu) = E_{T^z=-1} - E_{T^z=+1}$$



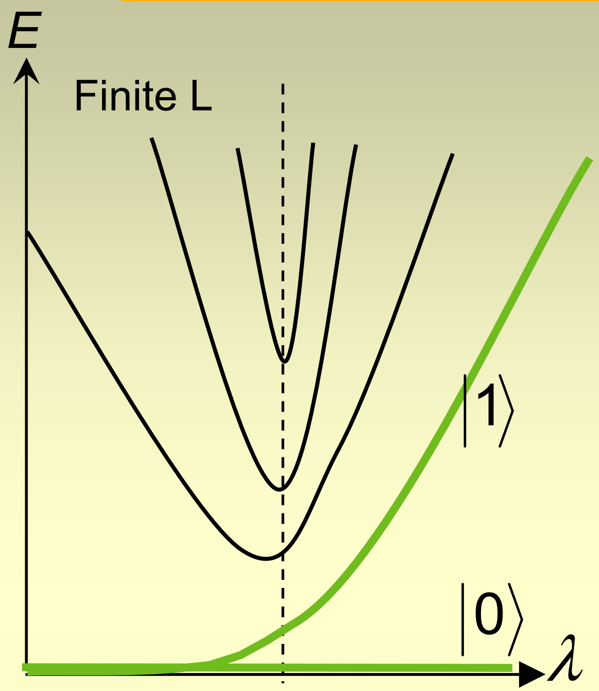


$$H_{eff}(\lambda,\mu) = -\tfrac{1}{2} f(\lambda) T^x - \tfrac{1}{2} g(\mu) T^z + \cdots$$

$\Rightarrow$ Allow any unitary rotation

Use $\lambda<1$ or $\mu<1$ (perturbative regime) :
L. B. Ioffe, *et al.*, Nature **415**, 503 (2002).

Quantum bit manipulation (II)



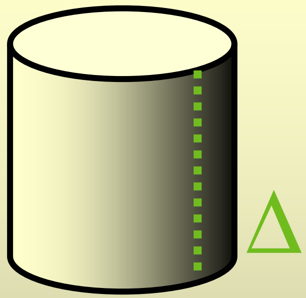
- Gap Δ~1/L close to the transition.

⇒ Requires a slow adiabatic (time~L) evolution to avoid transitions to higher levels close to the critical point.

To be compared to ~exp(L) if one stays in the perturbative regime [Ioffe *et. al.*, Nature 2002].

⇒ Requires a low temperature $k_B T \ll \Delta$ to avoid thermal excitations across the gap.

- Reading out - projection



$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

How to measure $|\alpha|^2$?

Switch on λ adiabatically to >>1 ⇒ minimizes N_Δ

$|N_\Delta \text{even}\rangle \rightarrow 0$ dimer along Δ *

$|N_\Delta \text{ odd}\rangle \rightarrow 1$ dimer along Δ *

⇒ No need to implement physically the non-local operator $T^x = (-1)^{N_\Delta}$

Summary

- Solvable quantum dimer model on kagome realizing a Z_2 liquid

- Toy model to investigate perturbations of a topological degeneracy

- Simple illustration of the manipulation of a topological qubit

Use of the phase transition to optimize the speed
and the protection against decoherence.