The Abdus Salam International Centre for Theoretical Physics

School and Workshop on Highly Frustrated Magnets and Strongly Correlated Systems: From Non-Perturbative Approaches to Experiments

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2D quantum magnetism and spin liquids

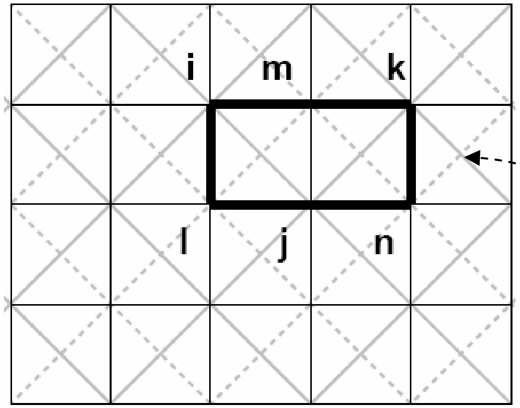
Grégoire Misguich

Service de Physique Théorique
Commissariat a l'Énergie Atomique (CEA)
Centre de Saclay, France

www-spht.cea.fr/pisp/misguich

Tutorial: Exact VBC ground-states in a toy model

Gellé, Lauchli, Kumar & Mila, arxiv:0704.2352v1



Sum over all 6-site rectangles

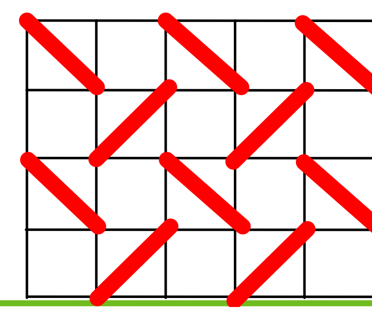
$$P^A_{i,j,k} = |\vec{S}_i + \vec{S}_j + \vec{S}_k|^2 - \frac{3}{4}$$

$$P^B_{l,m,n} = |\vec{S}_l + \vec{S}_m + \vec{S}_n|^2 - \frac{3}{4}$$

Projectors onto the states with total spin S=3/2

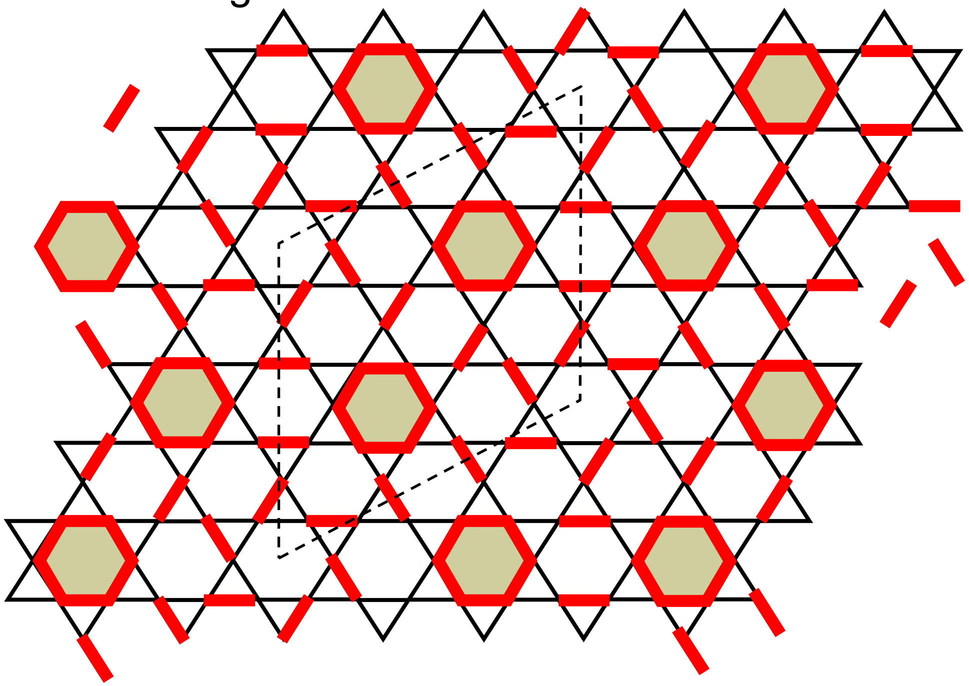
$$H_0 = \sum_{[i,j,k,l,m,n]} \frac{1}{4} P^A_{i,j,k} P^B_{l,m,n}$$

- Show that this singlet-product state : is an exact ground-state of H_0:



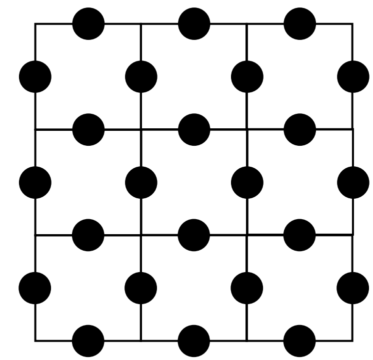
Tutorial – VBC stabilization by local resonances

- What is the ground-state degeneracy of the VBC below ? How many sites and bonds per unit cell ?
- Compare the variational energy of some arbitrary first-neighbor valence bond covering on the kagome lattice with that of the VBC below (with resonance around each shaded hexagon)
Hint : the ground-state energy of a periodic Heisenberg chain with 6 sites is E=-2.803



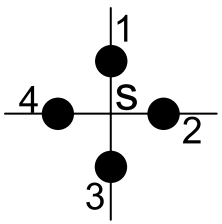
Such VBC phase has been proposed for the spin-½ Heisenberg model on the kagome lattice: see Marston & Zeng 1991; Nikolic & Senthil 2003; Singh & Huse 2007

Tutorial – Kitaev's model



Simplest spin model with (Z_2) fractionalization.
Elementary excitations are similar to that of a short-range RVB liquid
Kitaev arXiv 1997 (Annals of Phys. 2003).

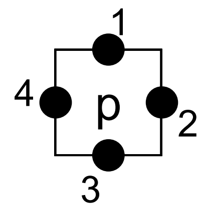
- Spin-1/2 model where the spins leaves on the bonds



$$H = -\sum_s A_s - \sum_p B_p$$

$$A_s = \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x$$

$$B_p = \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z$$

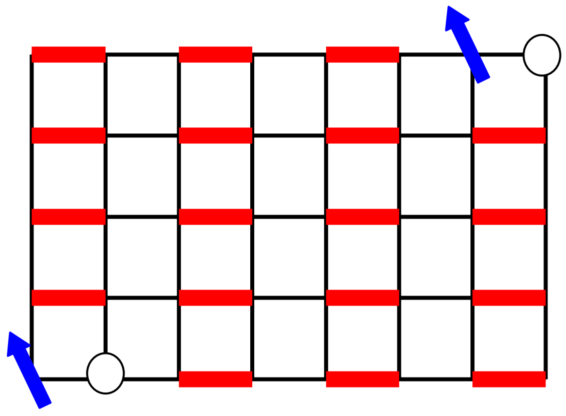


- Show that $[A_s,B_p]=0$
- Consider the equal amplitude superposition |0> of all the spin configurations (in the σ^z basis) with an *even* number of up down spins on each plaquette *p*.
  Show that |0> is a ground state of H
- We call "vison" a plaquette *p* with B_p=-1. Show that the number of "vison" is always even.
- We call "spinon" a vertex *s* with A_s=-1. Show that the number of "spinon" is always even.
- Construct some eigenstate of H with two remote "visons". Do the same with two "spinons".
- Show that the ground-state is two-fold degenerate on a cylinder. Hint: consider a product of σ^z operators on a loop winding around the cylinder.

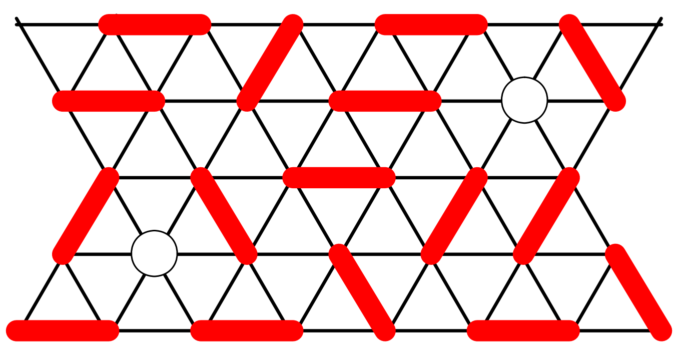
Sachdev & Vojta, cond-mat/0009202

- Consider two spin systems: one in a columnar VBC phase, and the other in a short-ranged RVB phase.
- We remove two spins (non-magnetic "impurities") at some fixed large distance from each other.
- In both cases, what are the lowest energy states (ground-state degeneracy ? Total spin ?)

- Valence-bond crystal

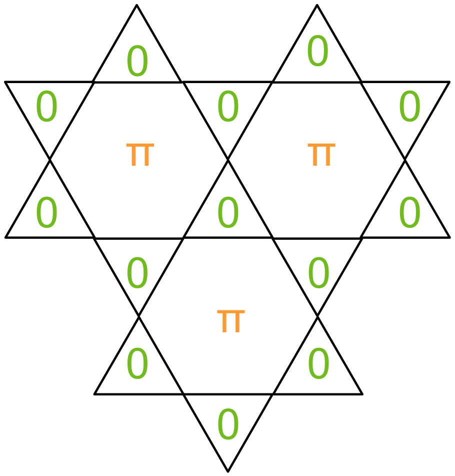


- Deconfined short-range RVB phase



Tutorial: a mean-field spin-liquid state on the kagome lattice

M. B. Hastings, Phys. Rev. B 63, 014413 (2000)
Y. Ran, M. Hermele, P. Lee & X.-G. Wen, Phys. Rev. Lett. 98, 117205 (2007)



$$\frac{J_{ij}}{8}\begin{bmatrix} \left\langle c_{i\uparrow}c^+_{j\uparrow} + c_{i\downarrow}c^+_{j\downarrow}\right\rangle & \left\langle c_{i\uparrow}c_{j\downarrow} - c_{i\downarrow}c_{j\uparrow}\right\rangle \\ \left\langle c^+_{i\downarrow}c^+_{j\uparrow} - c^+_{i\uparrow}c^+_{j\downarrow}\right\rangle & \left\langle c^+_{i\downarrow}c_{j\downarrow} + c^+_{i\uparrow}c_{j\uparrow}\right\rangle \end{bmatrix} = \begin{bmatrix} -\chi^+_{ij} & \eta_{ij}=0 \\ \eta^+_{ij}=0 & \chi_{ij} \end{bmatrix}$$

$$H_{MF} = \sum_{\langle ij\rangle} \chi_{ij}\left(c^+_{i\downarrow}c_{j\downarrow} + c^+_{i\uparrow}c_{j\uparrow}\right) + H.c$$

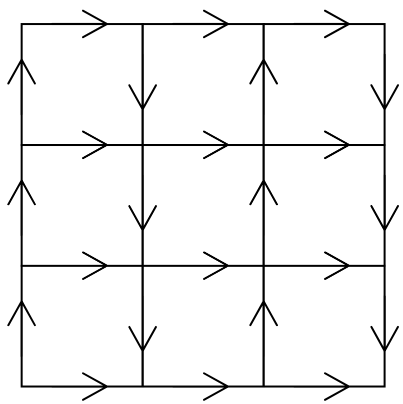
$$\chi_{ij} = \pm 0.221 \qquad \prod_{\text{triangle}} \chi_{ij} = 1 \qquad \prod_{\text{hexagon}} \chi_{ij} = -1$$

- Find some χ_{ij} such that the flux is π per hexagon, and 0 per triangle.
- Show that the invariant gauge group is U(1).

Tutorial: π-flux state

π-flux state

Affleck & Marston, Phys. Rev. B 37, 3774 (1988)
(equivalent to the "mixed" *s* + *id* RVB state of Kotliar & Liu, Phys. Rev. B 38, 5142 (1988))



$$U^0_{i\to j} = i\begin{bmatrix} u & 0 \\ 0 & u \end{bmatrix} \quad u \in \Re$$

$$\vec{A}^0_i = 0$$

$$H_{MF} = 2u \sum_{\langle i\to j\rangle} \left(i c^+_{\uparrow i} c_{\uparrow j} + \text{H.c} \right)$$

- Compute the SU(2) "flux" $U_{12}U_{23}U_{34}U_{41}$ piercing each square plaquette.
- Show that this state is translation invariant
- Show that invariant gauge group of this state is SU(2)
- Show that the spinon dispersion relation is:

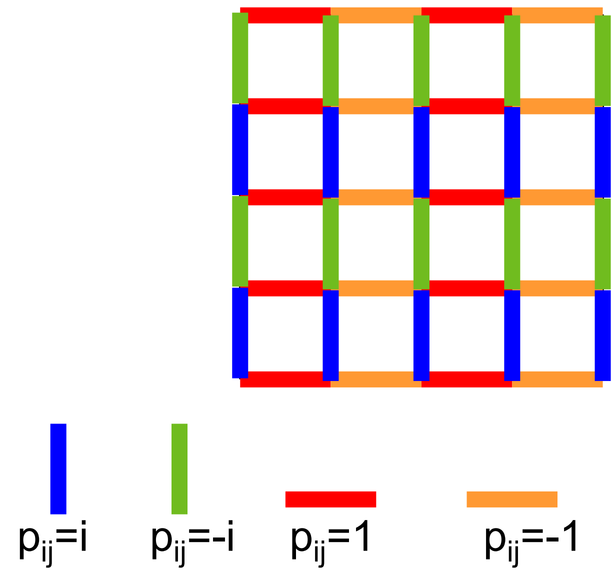
$$E_k = \pm 4u\sqrt{\sin^2(k_x) + \sin^2(k_y)}$$

Tutorial: valence-bond crystal order parameters

- Show that P is an order parameter for a columnar VBC

$$P = \frac{1}{N}\left\langle 0 \left| \left| \sum_{\langle ij \rangle} p_{ij} \vec{S}_i \cdot \vec{S}_j \right|^2 \right| 0 \right\rangle$$

$$= \frac{1}{N} \sum_{\langle ij \rangle} \sum_{\langle kl \rangle} p_{ij} \bar{p}_{kl} \underbrace{\left\langle 0 \left| \left(\vec{S}_i \cdot \vec{S}_j \right) \left(\vec{S}_k \cdot \vec{S}_l \right) \right| 0 \right\rangle}_{\text{4-spin correlation}}$$



- Find some order parameter(s) which can distinguish a plaquette and a columnar phases: