Wonders of E_{10} and $K(E_{10})$

Hermann Nicolai
MPI für Gravitationsphysik (AEI), Potsdam

Wonders of Gauge Theory and Supergravity
IHP and LPT-ENS, Paris, 23 - 28 June 2008

(mostly) based on work done in collaboration with:
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Main message of this talk:

Search for unification = search for symmetries

Most successful guiding principle of physics

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... and perhaps also for quantum gravity...

The BKL Paradigm

- Near a spacelike (cosmological) singularity, Einstein equations should simplify $\Rightarrow$ BKL decoupling: $\partial_x \ll \partial_t$?

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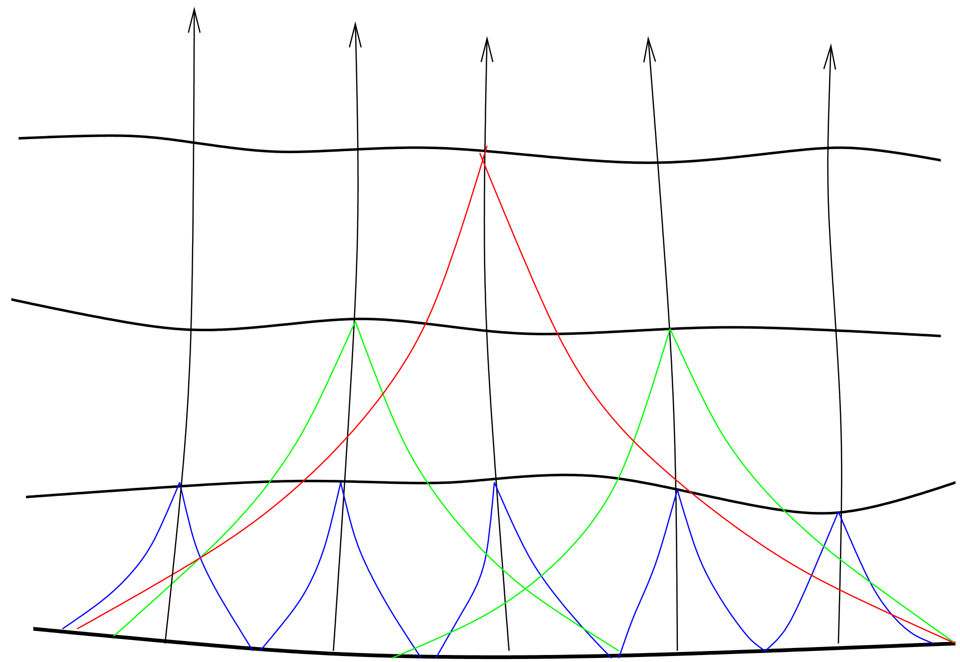

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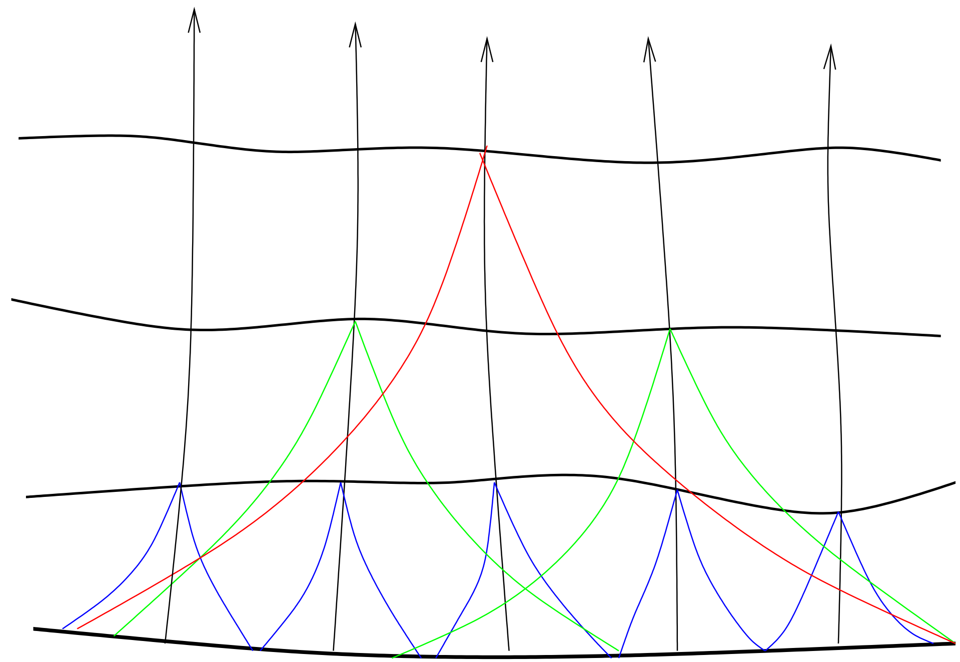

- Dimensional reduction to one (time) dimension $\to$ effective dynamics near singularity from gradient expansion? $\to$ billiards, chaotic oscillations, etc.

Another (old) paradigm

- Cosmological evolution as 'geodesic motion' in the moduli space of 3-geometries [Wheeler, DeWitt,...]:

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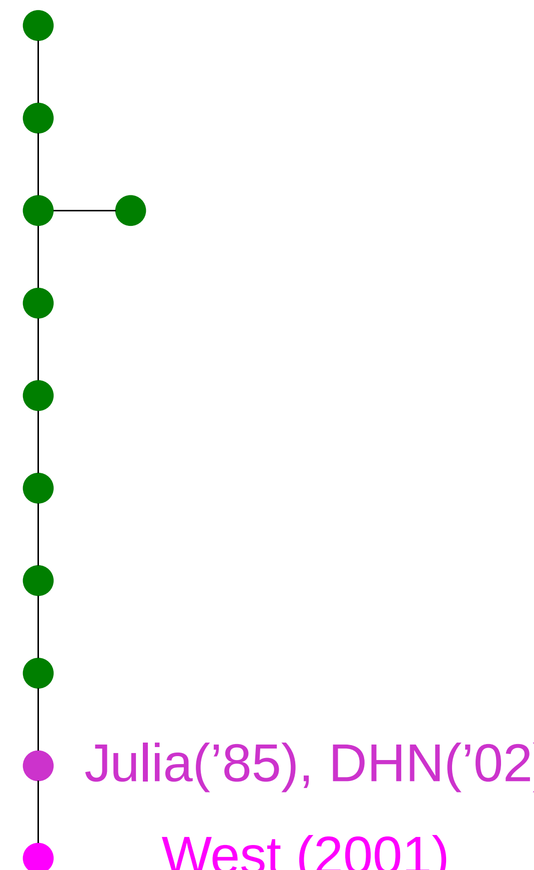
- Unification of space-time, matter and gravitation: configuration space $\mathcal{M}$ for quantum gravity should consistently incorporate matter degrees of freedom.

Hidden symmetries

Reduction of SUGRA$_{11}$ to $D = 11 - n$ [Cremmer, Julia (1979)]

n	Scalar Coset $E_n/K(E_n)$
1	$GL(1)/\mathbf{1}$
2	$GL(2)/SO(2)$
3	$SL(3) \times SL(2)/U(2)$
4	$SL(5)/SO(5)$
5	$SO(5,5)/SO(5) \times SO(5)$
6	$E_6/USp(4)$
7	$E_7/SU(8)$
8	$E_8/(Spin(16)/\mathbb{Z}_2)$
9	$E_9/K(E_9)$
10	$E_{10}/K(E_{10})$
11	$E_{11}/K(E_{11})$



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[Damour, Henneaux, hep-th/0012172; DHN, hep-th/0212256]

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motivates BASIC CONJECTURE: $\mathcal{M} = E_{10}/K(E_{10})$

Dynamics of supergravity (or some M theoretic extension)	Null geodesic motion on $E_{10}/K(E_{10})$ coset space

are equivalent! [DHN, hep-th/0207267]

SUGRA eqs. of motion + canonical constraints	∞-component geodesic eqn. and coset constraints

Definition of E_{10}

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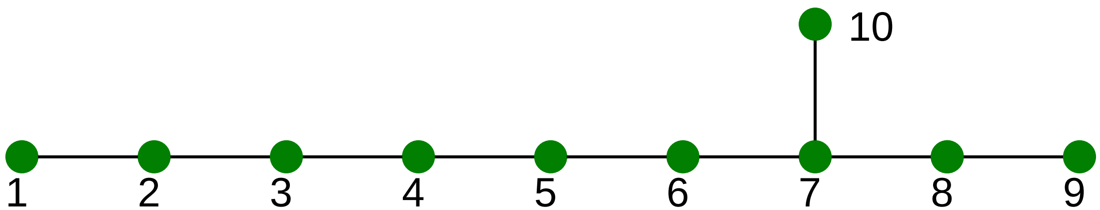
E_{10} is the Kac–Moody group with Kac–Moody Lie algebra $\mathfrak{g} \equiv \mathfrak{e}_{10}$ of rank 10 defined via the Dynkin diagram



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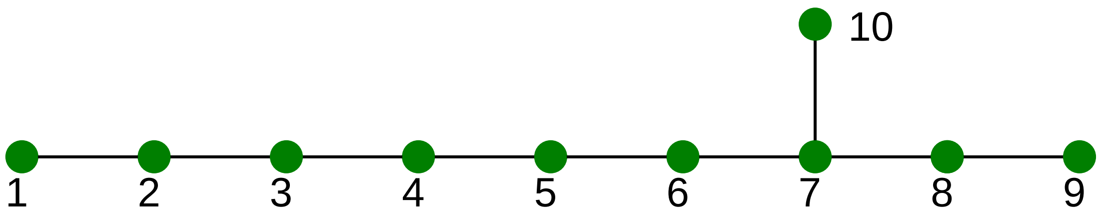

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Chevalley–Serre presentation: Generators h_i, e_i, f_i for $i = 1, \ldots, 10$ with relations

$$[h_i, h_j] = 0, \qquad [e_i, f_j] = \delta_{ij} h_i,$$

$$[h_i, e_j] = A_{ij} e_j, \qquad [h_i, f_j] = -A_{ij} f_j,$$

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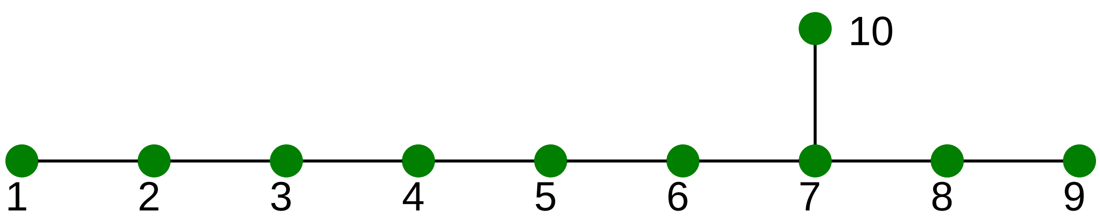

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h_i span Cartan subalgebra $\mathfrak{h}$; e_i and f_i: positive and negative simple root generators

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$$\mathfrak{g}_\alpha = \left\{ x \in \mathfrak{g} \,:\, [h,x] = \alpha(h)x \quad \text{for}\, h \in \mathfrak{h} \right\}$$

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- Triangular decomposition $\longrightarrow$ Computability

$$\mathfrak{g} = \mathfrak{e}_{10} = \mathfrak{n}_- \oplus \mathfrak{h} \oplus \mathfrak{n}_+ \quad , \quad \text{with } \mathfrak{n}_\pm := \bigoplus_{\alpha \gtrless 0} \mathfrak{g}_\alpha$$

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Fixed point set

$$\mathfrak{k}_{10} \equiv K(\mathfrak{e}_{10}) = \left\{ x \in \mathfrak{e}_{10} : \omega(x) \equiv -x^T = x \right\}$$

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However: $\mathfrak{k}_{10}$ is *not* a Kac-Moody algebra `[KN, hep-th/0506238]`

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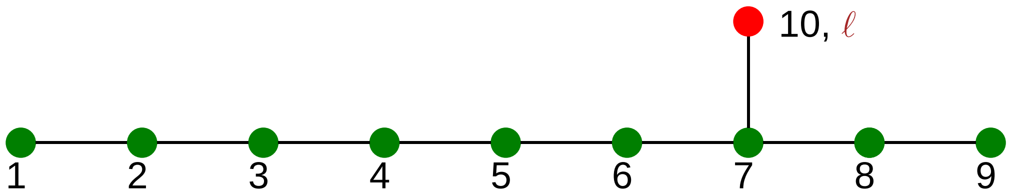

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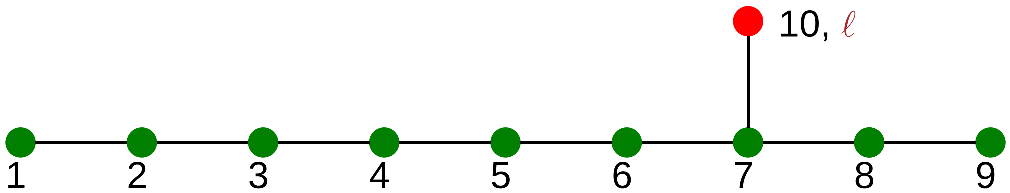

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ℓ	A_9 module	Tensor
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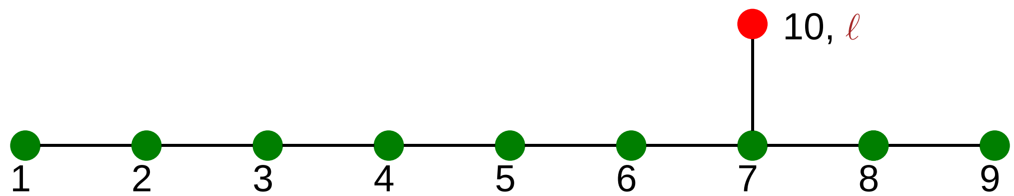

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These are just the representations corresponding to the bosonic fields of $D = 11$ SUGRA and their magnetic duals.

At level $\ell = 3$: dual graviton $h_{a_1 \dots a_8 | a_9}$ (with $h_{[a_1 \dots a_8 | a_9]} = 0$)

`[For more representations, see:  Fischbacher,N. hep-th/0301017]`

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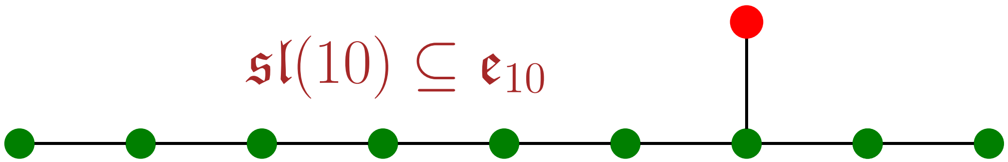

$D = 11$ SUGRA

[DHN; West 2002]

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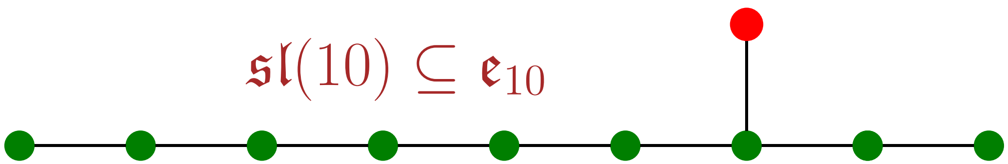

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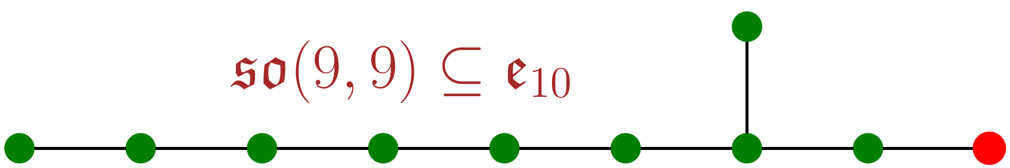

mIIA $D = 10$ SUGRA

[Kleinschmidt, Schnakenburg, West 2003]
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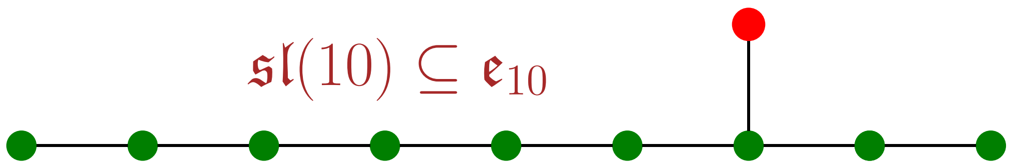

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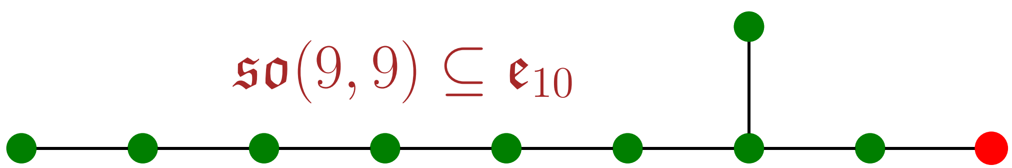

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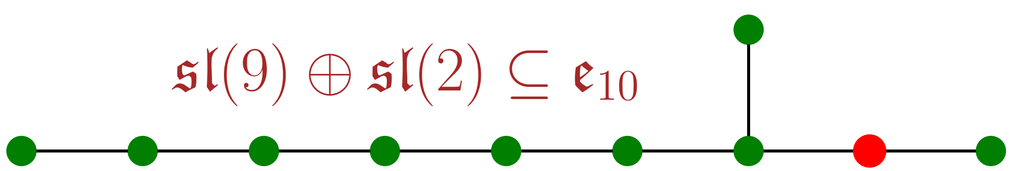
$\mathfrak{sl}(9) \oplus \mathfrak{sl}(2) \subseteq \mathfrak{e}_{10}$



IIB $D = 10$ SUGRA

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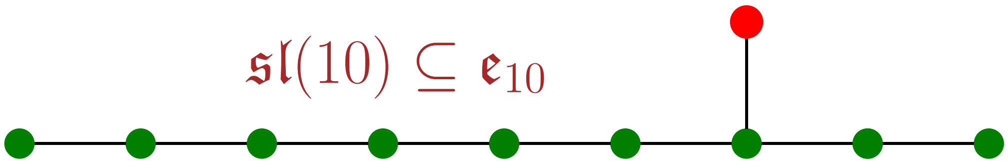

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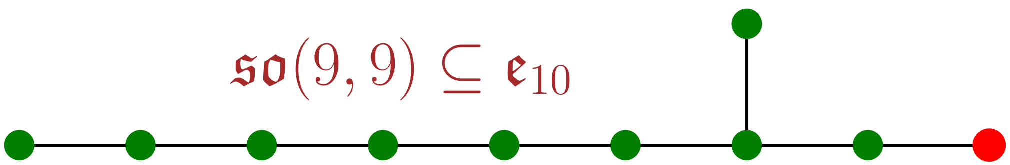

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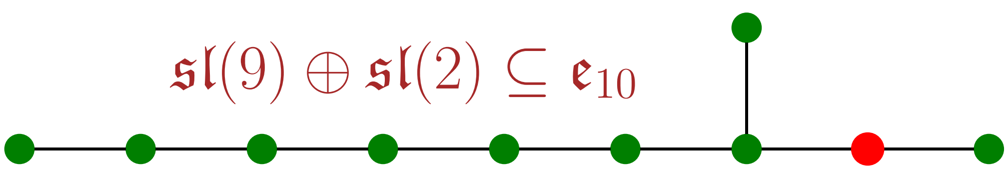

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These are the (maximal) low energy theories of the 'M-theory diagram', now all part of a single model.

Dynamics: bosonic Lagrangian

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Decompose Cartan form for $\mathcal{V}(t) \in E_{10}/K(E_{10})$

$$\partial_t \mathcal{V}\mathcal{V}^{-1}(t) = \mathcal{Q}(t) + \mathcal{P}(t) \quad , \quad \mathcal{Q} \in \mathfrak{k}_{10} \, , \, \mathcal{P} \in \mathfrak{e}_{10} \ominus \mathfrak{k}_{10}$$

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$\Rightarrow$ essentially unique coset Lagrangian ($n(t)$= lapse)

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Equations of motion: null geodesic on $E_{10}/K(E_{10})$

$$n\partial_t(n^{-1}\mathcal{P}) = \big[\mathcal{Q}, \mathcal{P}\big], \qquad \langle \mathcal{P} | \mathcal{P} \rangle = 0.$$

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With $\partial_t \mathcal{V}\mathcal{V}^{-1} = \sum_{\ell\geq 0} P^{(\ell)} * E^{(\ell)}$ (schematically) and truncation $P^{(\ell)} = 0$ for $\ell > 3 \;\Rightarrow$

Equations of motion up to $\ell = 3$ ($a, b = 1, \dots, 10$) [DHN; DN, hep-th/0410245]

$$
\begin{aligned}
n\mathcal{D}^{(0)}(n^{-1}P^{(0)}_{ab}) &= -\frac{1}{4}\big(P^{(1)}_{acd}P^{(1)}_{bcd} - \frac{1}{9}\delta_{ab}P^{(1)}_{cde}P^{(1)}_{cde}\big) \\
&\quad -\frac{1}{2\cdot 5!}\big(P^{(2)}_{ac_1\dots c_5}P^{(2)}_{bc_1\dots c_5} - \frac{1}{9}\delta_{ab}P^{(2)}_{c_1\dots c_6}P^{(2)}_{c_1\dots c_6}\big) \\
&\quad +\frac{4}{9!}\big(P^{(3)}_{ac_1\dots c_7|c_8}P^{(3)}_{bc_1\dots c_7|c_8} + \frac{1}{8}P^{(3)}_{c_1\dots c_8|a}P^{(3)}_{c_1\dots c_8|b} \\
&\qquad -\frac{1}{8}\delta_{ab}P^{(3)}_{c_1\dots c_8|c_9}P^{(3)}_{c_1\dots c_8|c_9}\big) \\
n\mathcal{D}^{(0)}(n^{-1}P^{(1)}_{abc}) &= -\frac{1}{6}P^{(2)}_{abcdef}P^{(1)}_{def} + \frac{1}{3\cdot 5!}P^{(3)}_{abcd_1\dots d_5|d_6}P^{(2)}_{d_1\dots d_6} \\
n\mathcal{D}^{(0)}(n^{-1}P^{(2)}_{a_1\dots a_6}) &= \frac{1}{6}P^{(3)}_{a_1\dots a_6 cde}P^{(1)}_{cde} \\
n\mathcal{D}^{(0)}(n^{-1}P^{(3)}_{a_1\dots a_8|a_9}) &= 0 \qquad \text{(with } P^{(3)}_{[a_1\dots a_8|a_9]} = 0\text{)}.
\end{aligned}
$$

This is a consistent truncation of $E_{10}/K(E_{10})$ coset dynamics: solutions of truncated theory are also solutions of the full theory.

Correspondence with SUGRA$_{11}$

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Bosonic $D = 11$ supergravity equations [Cremmer, Julia, Scherk 1978]

$$\begin{aligned} \mathcal{E}_{AB} &\equiv R_{AB} - \frac{1}{3} F_{ACDE} F_B{}^{CDE} + \frac{1}{36} \eta_{AB} F_{CDEF} F^{CDEF} = 0 \\ \mathcal{M}^{BCD} &\equiv D_A F^{ABCD} + \frac{1}{576} \epsilon^{BCDE_1 \dots E_8} F_{E_1 \dots E_4} F_{E_5 \dots E_8} = 0 \end{aligned}$$

and Bianchi identities: $D_{[A} F_{BCDE]} = R_{[AB\,C]D} = 0$

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Consider gauge fi xed (*à la* ADM) equations *at some fixed spatial point* $\mathbf{x}_0$:

- keeping all temporal and fi rst order spatial derivatives at $\mathbf{x}_0$
- zero-shift gauge: $E_M{}^A = \left(\begin{array}{c|c} N & 0 \\ \hline 0 & e_m{}^a \end{array} \right)$ and Coulomb gauge: $A_{tmn} = 0$
- Anholonomy coeffi cients $[\partial_b, \partial_c] = \tilde{\Omega}_{bc|a} \partial_a$ chosen traceless (in some neighborhood of $\mathbf{x}_0$) by exploiting *spatial* Lorentz group, i.e. $\Lambda_{ab} = \Lambda_{ab}(t, \mathbf{x})$ [???]
- Thus the standard ADM procedure leads to usual split into:

  – Dynamical equations: $\mathcal{E}_{ab} = \mathcal{M}_{abc} = D_{[0} F_{bcde]} = R_{[0a\,b]c} = 0$

  – Canonical constraints: $\mathcal{E}_{00} = \mathcal{E}_{0a} = \mathcal{M}_{0ab} = D_{[a} F_{bcde]} = R_{[ab\,c]d} = 0$

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Then with the identifi cation $n = Ne^{-1}$ and (r.h.s. always at fi xed spatial point $\mathbf{x} = \mathbf{x}_0$)

$$\mathcal{D}^{(0)} P^{(0)}_{ab} = R^{\text{time derivatives}}_{ab}$$

$$P^{(1)}_{abc} = N F_{0abc}$$

$$P^{(2)}_{a_1 \dots a_6} = -\frac{1}{4!} N \epsilon_{a_1 \dots a_6 b_1 \dots b_4} F_{b_1 \dots b_4}$$

$$P^{(3)}_{a_1 \dots a_8 | a_9} = \frac{3}{2} N \epsilon_{a_1 \dots a_8 bc} \tilde{\Omega}_{bc|a_9}$$

the two sets of dynamical equations coincide! (recall $P^{(3)}_{[a_1 \dots a_8 | a_9]} = 0 \Leftrightarrow \tilde{\Omega}_{ab|b} = 0$)

Dynamical equations for mIIA and IIB similarly from level decompositions w.r.t. fi nite dimensional subgroups $D_9 \equiv SO(9,9) \subset E_{10}$ and $A_8 \times A_1 \equiv SL(9) \times SL(2) \subset E_{10}$.

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Conserved E_{10} current $\mathcal{J} = n\mathcal{V}\mathcal{P}\mathcal{V}^{-1}$ ($\equiv$ Noether charge associated with global E_{10}):

$$\mathcal{J} = \frac{1}{9!} J_{(-3)}^{m_0|m_1\dots m_8} F_{m_0|m_1\dots m_8} + \frac{1}{6!} J_{(-2)}^{m_1\dots m_6} F_{m_1\dots m_6} + \frac{1}{3!} J_{(-1)}^{mnp} F_{mnp}$$
$$+ J_{(0)m}^{n} K^m{}_n + \frac{1}{3!} J_{(1)\,mnp} E^{mnp} + \frac{1}{6!} J_{(2)\,m_1\dots m_6} E^{m_1\dots m_6} + \dots$$

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Consider Sugawara-like ($\propto \mathcal{J} \otimes \mathcal{J}$) expressions [DKN, hep-th 0709.2691]

$$\mathfrak{L}_{(-6)}^{m_1\dots m_{10}\,;\,n_0|n_1\dots n_7} = J_{(-3)}^{n_0|m_1\dots m_8} J_{(-3)}^{m_9|m_{10} n_1\dots n_7}$$
$$\mathfrak{L}_{(-5)}^{m_1\dots m_{10};n_1\dots n_5} = J_{(-2)}^{n_1\dots n_4 m_1 m_2} J_{(-3)}^{m_3\dots m_{10}}$$
$$\mathfrak{L}_{(-4)}^{m_1\dots m_{10}\,;n_1 n_2} = \frac{21}{5} J_{(-2)}^{n_1 m_1\dots m_5} J_{(-2)}^{n_2 m_6\dots m_{10}} + J_{(-1)}^{n_1 m_1 m_2} J_{(-3)}^{n_2|m_3\dots m_{10}}$$

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(with appropriate antisymmetrizations) to re-express canonical constraints:

$$\mathfrak{L}_{(-6)}^{m_1\dots m_{10}\,;\,n_0|n_1\dots n_7} \propto \epsilon^{m_1\dots m_{10}} \epsilon^{n_1\dots n_7 pqr} R_{pq\,rn_0}$$ Bianchi (I)

$$\mathfrak{L}_{(-5)}^{m_1\dots m_{10};n_1\dots n_5} \propto \epsilon^{m_1\dots m_{10}} \epsilon^{n_1\dots n_5 p_1\dots p_5} D_{p_1} F_{p_2\dots p_5}$$ Bianchi (II)

$$\mathfrak{L}_{(-4)}^{m_1\dots m_{10}\,;n_1 n_2} \propto \epsilon^{m_1\dots m_{10}} \mathcal{M}^{0 n_1 n_2}$$ Gauss constraint

$$\mathfrak{L}_{(-3)}^{m_1\dots m_9} \propto \epsilon^{m_1\dots m_9 n} \mathcal{E}_{0n}$$ Momentum constraint

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  - – correct supermultiplets of SUGRA$_{11}$ and all $D < 11$ maximal supergravities
  - – in particular: self-duality of 5-form fi eld strength in IIB;
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- Fermions correctly described by R symmetry $K(E_{10})$.

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Thank you for your attention